

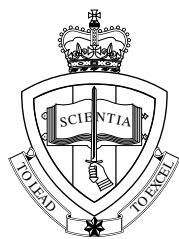
# Calculus on a TI-83/83+

Peter McIntyre

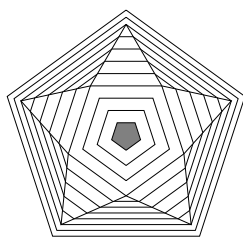
## Contents

1	Introduction	1
2	Basic Calculus Operations	2
3	A Classic Problem	8
4	Maximum and Minimum	11
5	Slope of a Graph	15
6	Discovering the Derivative by Exploration	17
7	Derivatives and the Shape of a Graph	19
8	Graphing Derivatives and Anti-derivatives	22
9	Rectangles, Area and the Definite Integral	25
10	Approximating Definite Integrals	27
11	Web Sites	31

*These materials may be copied for use by teachers and students for non-commercial educational purposes only.*



ADFA



CMA



T<sup>3</sup>

## Contact for more help

Peter McIntyre

School of Physical, Environmental  
and Mathematical Sciences  
UNSW@ADFA  
Canberra ACT 2600

Email: [p.mcintyre@adfa.edu.au](mailto:p.mcintyre@adfa.edu.au)  
Phone: (02) 6268 8896  
FAX: (02) 6268 8786

At [www.unsw.adfa.edu.au/pems/news/high\\_school/hsc\\_activities.html](http://www.unsw.adfa.edu.au/pems/news/high_school/hsc_activities.html)

- A variety of graphics-calculator activities for Years 9 and 10 — written as part of the CQTP Program.
- TI-83/83+ programs and program information.
- *Using the TI-83/83+* — an introduction to the basic operations, suitable for Years 8–12.
- *The Graphics Screen and Accuracy* — information to help you understand the graphical and numerical limitations of a graphics calculator.
- *Coordinate Geometry on a TI-83/83+* — basic commands and a variety of problems, suitable for Years 9 and 10.
- *Population Modelling* — a variety of problems from simple exponential growth to Leslie matrices and difference equations, covering Years 7–12.
- *Sequences and Series on a TI-83/83+* — basic commands and a variety of problems, suitable for Years 10–12.
- *Matrices on a TI-83/83+* — suitable for Years 11 and 12.
- *Complex Numbers on a TI-83/83+* — suitable for Years 11 and 12.
- *Introduction to Complex Numbers* — complex numbers from the beginning, covering the basic operations, but set in the context of complex numbers as a mathematical structure.
- *Programming a TI-83/83+* — suitable for teachers and keen students.

All the programs listed in these notes can be found at the above web site. You will need *TI Connect*, available free at

[education.ti.com/us/product/accessory/connectivity/features/software.html](http://education.ti.com/us/product/accessory/connectivity/features/software.html)

(click on *download* at left to download) and a special cable (costs money) to download programs from your computer to your calculator.

# 1 Introduction

Graphics calculators lend themselves very nicely to demonstrating the visual aspects of Calculus — graphs of functions, tangent lines, areas under curves, etc — as well as to calculating numerically (approximating) many of the quantities that arise — derivatives and definite integrals, maximum and minimum values, etc.<sup>1</sup>

They can be used at a number of levels.

- As a basic graph plotter — what does the graph of  $y = e^x$  look like?
- To investigate ‘what if’ questions, for example what happens if you change the parameters  $a$  and  $b$  in the equation  $y = ae^{bx}$ ?
- To do (numerically) many of the basic calculations in Calculus, such as finding the slope at a point on a graph, definite integrals, maxima and minima, etc.
- To illustrate graphically, perhaps by way of a program, some of the concepts of Calculus. Two examples are showing how a secant line tends to a tangent line in the appropriate limit and how we can approximate the area under a graph by the areas of some rectangles. With sufficient ingenuity, almost anything can be done here, the only limitation being the small screen of the calculator.
- To automate, using the built-in operations or programs, some of the calculations that arise in Calculus: numerical integration methods, solving differential equations numerically and so on.

At a more mundane level, graphics calculators are fun. Students pick up the operations very quickly (much faster than teachers), and if you can’t get your students to use a graphics calculator, there are heaps of games to tempt them.<sup>2</sup>

The other good news is that there are lots of resources available, many free on the Web, as well as a rapidly increasing number of books on using graphics calculators in almost every aspect of Mathematics and Science.

Getting started is always the hardest, especially when you have to modify or write new courses, but the experience at ADFA and most other schools and universities at which graphics calculators have been used for a while, is that graphics calculators should not just be an add-on to a course, but should be integrated fully, including their use in tests and exams. This raises some issues, most of which are resolvable. You might like to read for example *Graphics calculators in the mathematics curriculum: Integration or differentiation?* by Jen Bradley, Barry Kissane and Marian Kemp about their experiences in WA.<sup>3</sup>

At UNSW@ADFA, we have been using TI calculators (TI-83+s at present) in our first-year courses since 1993 and have come to appreciate their worth in learning Mathematics.

---

<sup>1</sup>There are, of course, ‘calculators’ that go even further and do things symbolically.

<sup>2</sup>The TI-83 runs an excellent version of Tetris, and there are many quite sophisticated games on the Web.

<sup>3</sup>This paper is available at [wwwstaff.murdoch.edu.au/~kissane](http://wwwstaff.murdoch.edu.au/~kissane) under Publications. There are a number of other interesting papers here too.

## 2 Basic Calculus Operations

### 1. Graph $f(x) = \sin(2x)$ for $0 < x < \pi$

Note that the graphics and table keys are the top row of keys.

- Press **Y=**: set  $Y_1 = \sin(2X)$ . The independent variable  $X$  is the **X, T,  $\theta$ , n** key in the third row of keys.

Note the highlighted = sign, which means the function will be plotted when you press **GRAPH**. Move the cursor over the = sign and press **ENTER** to toggle the function off/on.

```

Plot1 Plot2 Plot3
\Y1=sin(2X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=

```

- Press **MODE** (*second row of keys*).
  - *Radian*: highlight with the cursor and press **ENTER**.
  - **2nd** **QUIT** to return to the home screen (or just press the next key).

```

Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^ $\theta$ i
Full Horiz G-T

```

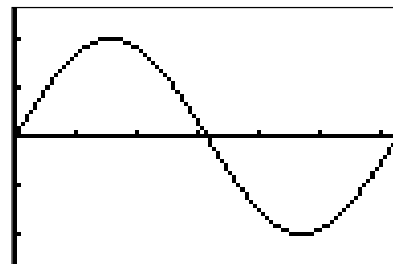
- Press **WINDOW**: specify the viewing window.
  - For  $X$ , suitable values here are:  
 $Xmin = 0$     $Xmax = \pi$     $Xscl = 0.5$ .  
 Use **ENTER** or the down arrow to move between values.  
 $\pi$  is **2nd**  **$\wedge$** .
  - $Xscl$  is the distance between tick marks on the  $X$  axis (cosmetic only: 0 gives no tick marks).
  - Suitable  $Y$  values here are:  
 $Ymin = -1.3$     $Ymax = 1.3$     $Yscl = 0.5$ .
  - Note the difference between the blue subtract key **-** and the white change-sign key **(-)**.
  - You could also use *ZFit* (**ZOOM** **0**), which calculates appropriate  $Y$  values for the function(s) and  $X$  values you set.

```

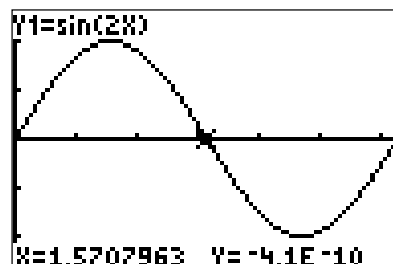
WINDOW
Xmin=0
Xmax=3.1415926...
Xscl=.5
Ymin=-1.3
Ymax=1.3
Yscl=.5
Xres=1

```

- Press **GRAPH**: graph the function.



- Press **TRACE**: move the cursor along the graph with the left- or right-arrow keys; the coordinates of the point on the graph are shown at the bottom.



- **TABLE**

Set the table 'WINDOW' using **TBLSET**:  
 $TblStart = 0$   $\Delta Tbl = 0.5$  *Auto Auto*.  
 Select *Auto* with the cursor and **ENTER**.



Press **TABLE**: scroll down in either column, up past the top in the X column.

Move the cursor to the Y1 column to see more digits in the Y value.

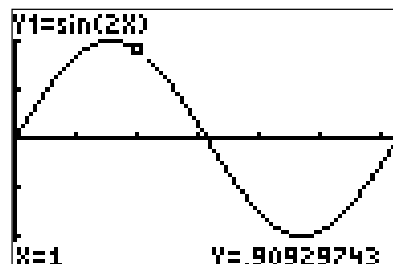
X	Y1	
0	0	
.5	.84147	
1	.9093	
1.5	.14112	
2	-.7568	
2.5	-.9589	
3	-.2794	

X=0

## 2. Estimate $f(1)$

- **On the graph**

- Press **TRACE**.
- Type in the X value, **1** **ENTER**, to move to the desired point on the graph. Note the coordinates at the bottom of the screen.
- Alternatively, use the left/right arrows to move the cursor along the curve (but note the problem when trying to reach  $X=1$ ).
- The up/down arrows move between functions if there is more than one graphed.



- On the home screen

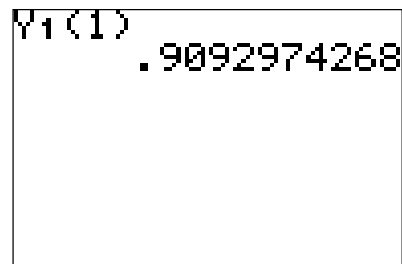
- Type in  $Y_1(1)$   $\boxed{\text{ENTER}}$ .

- $Y_1$  is in the  $\boxed{\text{VARS}}$  Y-VARS Function menu:

- $\boxed{\text{VARS}}$   $\boxed{\rightarrow}$   $\boxed{1}$   $\boxed{1}$ .

- You can't just type  $\boxed{Y}$   $\boxed{1}$ .

- *Answer:*  $f(1) = 0.90930$ , rounded to 5 decimal places.
- *Accuracy:* 'at least five accurate digits' (TI-83 Guidebook, B11).



Y1(1)  
.9092974268

3. Estimate  $f'(1)$ 

- On the graph

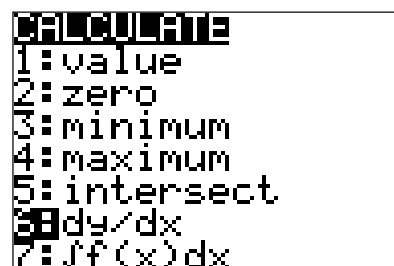
- Press  $\boxed{\text{GRAPH}}$ .

- Select  $dy/dx$  in the CALC menu ( $\boxed{2nd}$   $\boxed{\text{TRACE}}$ ).

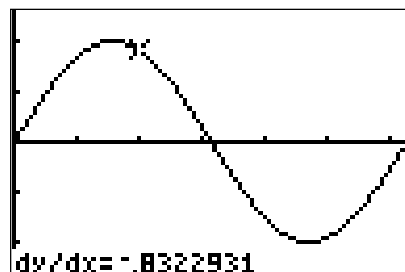
- Type in the X value:  $\boxed{1}$   $\boxed{\text{ENTER}}$ .

- Alternatively, use the arrow keys to move the cursor to a point on the curve and press  $\boxed{\text{ENTER}}$ .

- *Tangent* in the  $\boxed{\text{DRAW}}$  menu draws the tangent at the point specified (as we did for  $dy/dx$ ) and gives its equation. You may have to adjust the number of decimal places displayed (using the second line in the  $\boxed{\text{MODE}}$  menu) so you can see the whole equation.



CALCULATE  
1: value  
2: zero  
3: minimum  
4: maximum  
5: intersect  
6: dy/dx  
7:  $\int f(x) dx$



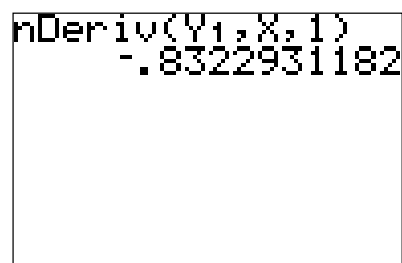
- On the home screen

- $nDeriv(Y_1, X, 1)$ .

- $nDeriv$  is in the  $\boxed{\text{MATH}}$  menu:  $\boxed{\text{MATH}}$   $\boxed{8}$ .

- The X is the variable we are differentiating  $Y_1$  with respect to.

- *Answer:*  $f'(1) = -0.83229$ , rounded to 5 decimal places.
- *Accuracy:* can be adjusted in  $nDeriv$  by an optional fourth argument. The default value for this argument is  $10^{-3}$ .



nDeriv(Y1, X, 1)  
-.8322931182

4. Estimate  $\int_0^\pi \sin(2x) dx$

- On the graph

- Press `GRAPH`.

- Select  $\int f(x) dx$  in the `CALC` menu.

Type in the lower and upper integration limits when prompted, followed each time by `ENTER`.



- On the home screen

- `fnInt(Y1, X, 0,  $\pi$ )`.

`fnInt` is in the `MATH` menu: `MATH` `9`.

- *Answer*:  $\int_0^\pi \sin(2x) dx = 0.00000$ , rounded to 5 decimal places. The  $E^{-14}$  stands for *times*  $10^{-14}$ .

- *Accuracy*: (claimed)  $10^{-5}$  — can be set in `fnInt` by an optional fifth argument.

```
fnInt(Y1,X,0, $\pi$ )
-5.1379602E-14
```

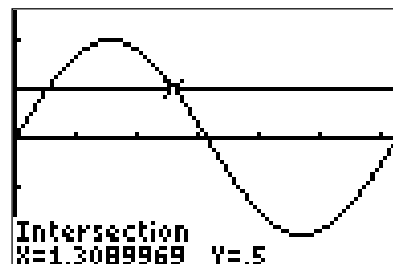
5. Solve  $\sin(2x) = 0.5$  for  $0 \leq x \leq \pi$

- On the graph

- Graph `Y1` and `Y2 = 0.5`.

- Use `intersect` in the `CALC` menu: press `ENTER` to select each curve; move the cursor to provide the *guess* or type in an `X` value and press `ENTER`. Your *guess* will determine which solution you find if there are several (as here).

If you have more than two curves, select the ones you want with the up/down arrows before pressing `ENTER`.



- From the home screen

- Use the interactive *Solver*: **MATH** **0**.

Enter the equation as

$$0 = \sin(2x) - 0.5$$

and press **ENTER**. Set the bounds to  $\{0, \pi\}$ , enter a guess for X (this will determine which of the two possible values you find) and with the cursor on X, press **SOLVE** (**ALPHA** **ENTER**).

- You can also use *solve* in the **CATALOG**.

- *Answer*: the curves intersect at  $x = 0.26180$  and  $x = 1.30900$ , both rounded to 5 decimal places.
- *Accuracy*: (claimed)  $10^{-5}$ .

```
EQUATION SOLVER
eqn:0=sin(2X)-0.
5
```

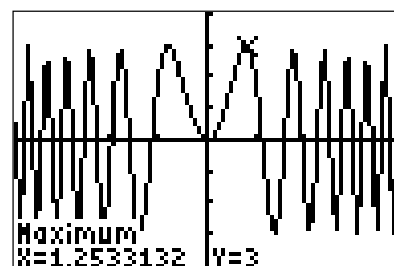
```
sin(2X)-0.5=0
X=1
bound=(0,π)
```

```
sin(2X)-0.5=0
▪ X=1.3089969389...
bound=(0,3.141...
▪ left-rt=0
```

## 6. Find the value of the first positive maximum of $3 \sin(x^2)$

- On the graph

- Set  $Y_1 = 3 \sin(X^2)$ . Turn off or clear all other functions. Press **ZOOM** **7** (*ZTrig*) to obtain standard trig axes and plot the function.
- Select *maximum* in the **CALC** menu. The calculator asks for a left bound, which you select by moving the cursor somewhere to the left of the maximum you want (or by typing in a value) and pressing **ENTER**. Similarly with the right bound and the guess. The guess doesn't have to be spot on. Use the bounds and the guess to pick out the (local) maximum you want.
- Press **2nd** **QUIT** and type  $2X^2$  **ENTER** on the home screen. You should recognise the first 6 digits of the number.

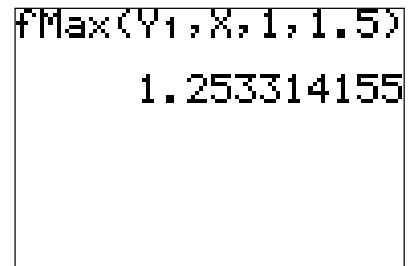


- **On the home screen**

- $fMax(Y_1, X, 1, 1.5)$  in the **MATH** menu: the last two inputs are the bounds for the search.

- *Answer*: the maximum value of 3 occurs at  $x = 1.25331$ , rounded to 5 decimal places.

- *Accuracy*: (claimed)  $10^{-5}$  — can be set in  $fMax$  by an optional fifth argument.



```
fMax(Y1,X,1,1.5)
1.253314155
```

The **CALC** operations *minimum* and *zero* work in the same way as *maximum*.

### 3 A Classic Problem

A hare and tortoise compete in a one-kilometre race. The distance each competitor has travelled from the starting point is given by a formula. In time  $t$  **minutes**, the distance in **metres** travelled by the hare is given by  $H(t) = \frac{500}{3}(2\sqrt{t} + \sqrt[3]{t})$ , while the distance in **metres** travelled by the tortoise is given by  $T(t) = 100t + 250\sqrt{t}$ .

Press  $\boxed{Y=}$  and enter the formulas for  $H$  and  $T$ . You have to use  $X$  ( $\boxed{X,T,\theta,n}$ ) as the independent variable. The cube root is  $\boxed{\text{MATH}} \boxed{4}$ .

Set your  $\boxed{\text{WINDOW}}$  so that the two graphs go from the bottom left to the top right of the screen. *Hints:* The race takes about 5 minutes. How far is the race?

If you select *Simul* in the  $\boxed{\text{MODE}}$  menu of your TI-83 before graphing, you will get a real-time view of the race. Choosing the graph style of a circle with a tail<sup>4</sup> for each competitor makes it even better.

Answer the following questions, writing down the steps you took. Plotting the lines  $Y_3 = 500$ ,  $Y_4 = 1000$  and using *intersect* in the  $\boxed{\text{CALC}}$  menu will be helpful. You may need to increase  $Y_{max}$  when using *intersect* so that the function formulas do not obscure the point you are interested in.

1. Who gets to the halfway point first? How long does it takes them? Verify your answer algebraically.
2. What is the time and distance at which the two runners are neck and neck?
3. Who wins the race, by what time margin and by what distance margin?



<sup>4</sup>Press  $\boxed{Y=}$ , move the cursor to the left of  $Y_1$  and press  $\boxed{\text{ENTER}}$  a couple of times. Repeat for  $Y_2$ .

## Teacher's Notes

The questions in this version have been written in general terms deliberately for a good class. For a less-advanced class, students may need to be led a little through each question: *What equation do we need to solve to answer this question? What does this mean about the graphs of each side of the equation? How do we solve this equation on the calculator? and so on.*

Press  $\boxed{Y=}$  and put the equation for the hare in  $Y_1$  and that for the tortoise in  $Y_2$ . Watch brackets here. You might like to discuss with the class how to write the formulas in a suitable form for the calculator. Time  $t$  becomes  $X$  on the calculator.

```

Plot1 Plot2 Plot3
\Y1=500/3*(2√(X)
+3√(X))
\Y2=100X+250√(X)

\Y3=
\Y4=
\Y5=

```

Then set the  $\boxed{\text{WINDOW}}$ . Discuss first with the class what each axis represents and suitable scales. The  $Y$  axis is distance in metres, so  $0 < Y < 1000$ . The winner is then the competitor whose graph first reaches the top of the screen (providing *Simul* is set in  $\boxed{\text{MODE}}$ ).

```

WINDOW
Xmin=0
Xmax=5
Xscl=1
Ymin=0
Ymax=1000
Yscl=100
Xres=1

```

The time ( $X$ ) scale has to be guessed. The race takes a little less than 5 minutes, so  $0 < X < 5$  gives a good view. Set  $Xscl$ , the distance between the tick marks on the  $X$  axis to 1 and  $Yscl$  to 100. If either of these is too small, you will get a double line for the axis.

```

Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T

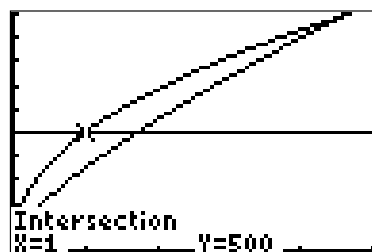
```

1. From the graph (use  $\boxed{\text{TRACE}}$  and the up/down arrows to see which graph is which), the hare clearly reaches the halfway point (500 m) first.



To find how long the hare took, solve  $H(t) = 500$  for  $t$ : set  $Y_3 = 500$  and find the intersection of  $Y_1$  and  $Y_3$  using *intersect* in the  $\boxed{\text{CALC}}$  menu.

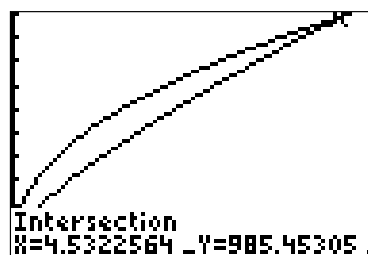
After you select *intersect*, the cursor will be on  $Y_1$ : press  $\boxed{\text{ENTER}}$  to select it. The cursor will move to  $Y_2$ : press the down-arrow key to move it to  $Y_3$  and press  $\boxed{\text{ENTER}}$  to select it. Move the cursor to somewhere near the intersection and press  $\boxed{\text{ENTER}}$  to provide a guess.



The value for  $t$  is 1 minute, a value we can confirm algebraically to be exact by substituting  $t = 1$  into the equation for the hare. Note that it is easy to **verify** that  $t = 1$  is a solution, but tricky to **solve**  $H(t) = 500$  algebraically (it turns into a cubic equation).

*The hare reaches the halfway point first in a time of 1 minute.*

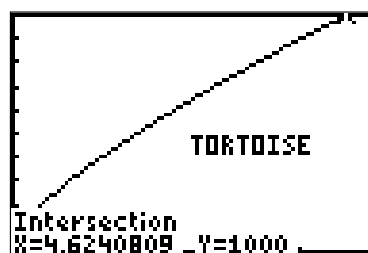
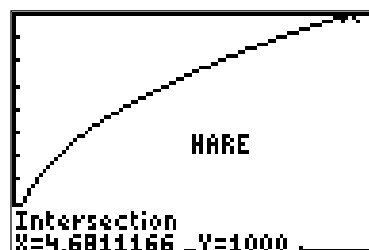
2. To find when they are neck and neck, we have to solve  $H(t) = T(t)$ , that is find the intersection of  $Y_1$  and  $Y_2$  (algebraically, this turns into a quartic equation). We obtain, using *intersect*,  $t = 4.53$  minutes and distance equal to 985 m (both to 3 significant digits). It might be useful to increase *Ymax* or *Zoom In* (**ZOOM** **2**) on this part of the graph to see the two curves more clearly.



*The hare and tortoise are neck and neck after about 4.53 minutes or about 4 minutes 32 seconds, at a distance of about 985 metres from the start.*

3. To find the winner, we have to determine the time at which each competitor reaches the finish (1000 m).

Setting  $Y_4 = 1000$ , we find the hare finishes at  $t = 4.681$  minutes (intersection of  $Y_1$  and  $Y_4$ ) and the tortoise finishes at  $t = 4.624$  minutes (intersection of  $Y_2$  and  $Y_4$ ).



To find the distance margin, calculate  $H(4.624)$ , the position of the hare when the tortoise finishes:  $H(4.624) \equiv Y_1(4.624) = 994.45$  m, rounded to 5 significant digits.

*The tortoise wins the race by a margin of 0.057 minutes or 3.42 seconds. The distance margin is 5.55 m.*

## Using the Newton-Raphson Method

The problems here are also a good application of the Newton-Raphson method for finding the zero of a function. For this, it is useful to have a program.

The program NEWTON for the TI-83/83+ can be downloaded at [www.ma.adfa.edu.au](http://www.ma.adfa.edu.au) under *High School and College Activities*. The program finds the zero of a graphed function, given an initial guess provided by the position of the cursor on the graph.

The program finds zeroes of the function in  $Y_1$ , so it is useful to put the hare and tortoise equations in  $Y_2$  and  $Y_3$ .  $Y_1$  can then be defined as  $Y_1 = Y_2 - 500$  to find when the hare reaches halfway;  $Y_1 = Y_2 - Y_3$  to find when they are neck and neck;  $Y_1 = Y_2 - 1000$  to find when the hare finishes and  $Y_1 = Y_3 - 1000$  to find when the tortoise finishes.

## 4 Maximum and Minimum

When manufacturers are designing their packaging, they must keep in mind the amount of product that has to fit inside and the amount of material it will take to make the package. Consider the humble soft-drink can. The standard volume is 375 mL or 375 cm<sup>3</sup>. Any number of cans can be designed that will hold this volume of liquid, but they will vary in shape and therefore in the amount of material needed to make the can (and therefore cost).

The formula for the volume of a cylinder  $V$  in terms of radius  $r$  and height  $h$ , is

$$V = \pi r^2 h.$$

Rearrange the volume formula to make  $h$  the subject and let the volume be 375 cm<sup>3</sup>. *What are the units of  $r$  and  $h$ ?*

$h =$

Enter this formula as  $Y_1$ , with  $X$  as radius  $r$ .

As a check, enter the volume formula:  $Y_2 = \pi X^2 Y_1$ .

$Y_1$  is in the **[VARS]** Y-VARS Function menu.

```

Plot1 Plot2 Plot3
\Y1=375/(piX^2)
\Y2=piX^2Y1
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=

```

Press **[TBLSET]** (**[2nd]** **[WINDOW]**).

Set  $TblStart = 1$  and  $\Delta Tbl = 1$ .

Press **[TABLE]** (**[2nd]** **[GRAPH]**).

*Do you get the correct value for the volume in  $Y_2$ ?*

```

TABLE SETUP
TblStart=1
deltaTbl=1
Indent:  Auto Ask
Depend:  Auto Ask

```

Write down the formula for the surface area of a cylinder, including the ends.

SA =

The surface area determines the amount of material needed to make the can. *Why?* Enter the formula for surface area in  $Y_2$  in terms of  $X$  (radius) and  $Y_1$  (height).

$Y_2 =$

View the table of values again. *What do you notice about the values of the surface area?*

Use **[TBLSET]** to set new starting values and smaller increments to find the minimum surface area and corresponding radius (radius accurate to 1 decimal place).

Minimum SA =

Radius =

Now graph the surface area as a function of radius, and use *minimum* in the CALC menu to find the minimum.

*Draw your graph here.*

Write down your values for the radius, height, surface area and circumference of the can when the surface area is a minimum. *Do these values seem reasonable?*

*How does this compare with a can of soft drink? Why the differences? What about other cans?*

You might like to read the article *The Best Shape for a Tin Can* by P.L. Roe, either in *The Mathematical Gazette* 75, 472 (1991) or reprinted in *The College Mathematics Journal* 24, 233 (1993).

### Teacher's Notes

This is an activity from: *Integrating the Graphics Calculator into Year 9 and Year 10 of the Victorian Mathematics CSF*, Teachers Teaching with Technology (T<sup>3</sup>), 1998, modified somewhat. There are a number of maximum/minimum activities for all years that work in a similar way.

The height of the cylinder is given by  $Y_1 = 375/(\pi X^2)$ , where  $X$  is the radius  $r$ . The brackets are essential here because of the order in which the calculator does the different operations.

As a check, enter the volume formula  $Y_2 = \pi X^2 Y_1$ .

Press **TABLE**.

X	Y <sub>1</sub>	Y <sub>2</sub>
1	119.37	375
2	29.842	375
3	13.263	375
4	7.4604	375
5	4.7746	375
6	3.3157	375
7	2.436	375

X=1

The total volume of the metal used to make the can, assuming the walls are of uniform thickness, is just the surface area times the thickness. Minimum surface area therefore means minimum volume of metal.

The surface area of a cylinder, including the ends, is given by

$$SA = 2\pi r^2 + 2\pi r h = 2\pi r(r + h).$$

Plot1	Plot2	Plot3
Y <sub>1</sub>	375/(πX <sup>2</sup> )	
Y <sub>2</sub>	2πX(X+Y <sub>1</sub> )	
Y <sub>3</sub>	=	
Y <sub>4</sub>	=	
Y <sub>5</sub>	=	
Y <sub>6</sub>	=	
Y <sub>7</sub>	=	

View the table of values again. What do you notice about the values of the surface area?

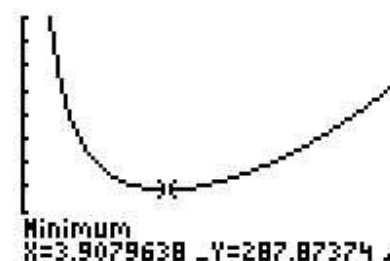
The surface area decreases then increases as the radius increases. There is a (local) minimum.

X	Y <sub>1</sub>	Y <sub>2</sub>
1	119.37	756.28
2	29.842	400.13
3	13.263	306.55
4	7.4604	288.03
5	4.7746	307.08
6	3.3157	351.19
7	2.436	415.02

X=1

With  $TblStart = 3$  and  $\Delta Tbl = 0.1$ , we find a radius of 3.9 cm for minimum surface area.

Alternatively, one can view the graph of surface area versus radius and use *minimum* in the CALC menu to find the minimum. The figure has a WINDOW of  $[0, 10, 2] \times [0, 1000, 100]$ .



Again, we obtain a value of  $r = 3.9$  cm (accurate to 1 decimal place) for the radius giving minimum surface area.

If your students have sufficiently developed calculus skills, they could prove algebraically that the global minimum lies at  $r = \sqrt[3]{375/2\pi} \approx 3.9$ . More generally, for a given volume  $V$ , it is not too hard to show that  $h = 2r$  (height = diameter) for the least surface area.

*Is this a reasonable answer for the radius?*

Other factors may have to be taken into account such as what circumference is comfortable for the average human hand, the wastage of material when cutting the ends and the cost of making the joins.

$$\text{radius } r = \sqrt[3]{\frac{375}{2\pi}} \approx 3.9 \text{ cm} \quad \text{height } h = \sqrt[3]{\frac{1500}{\pi}} \approx 7.8 \text{ cm} \quad \text{ratio } \frac{h}{r} = 2$$

$$\text{surface area } \approx 288 \text{ cm}^2 \quad \text{circumference } \approx 24.6 \text{ cm}$$

*How does this compare with a can of soft drink?*

A soft-drink can has a radius of 3.25 cm and a height of 13 cm:  $h/r = 4$ . Its surface area is about 332 cm<sup>2</sup> and circumference 20.4 cm. The article *The Best Shape for a Tin Can*<sup>5</sup> by P. L. Roe, either in *The Mathematical Gazette* 75, 472 (1991) or reprinted in *The College Mathematics Journal* 24, 233 (1993) goes into why there might be differences between the theory here and the actual values. A good example of mathematical modelling.

---

<sup>5</sup>A copy of this article is available from Peter McIntyre.

## 5 Slope of a Graph

The program DIFFQPLT illustrates graphically how the difference quotient (the slope of the secant line) approaches the derivative (the slope of the tangent line) at a point on a graph.

- Put the function  $f(x) = e^x$  in Y1. We wish to find the slope of the graph of  $f$  at  $x = 1$ .

```

Plot1 Plot2 Plot3
Y1=e^(X)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

```

- Run the program — press `PRGM`, press the number against DIFFQPLT (or scroll down and press `ENTER`) and press `ENTER` to start the program.

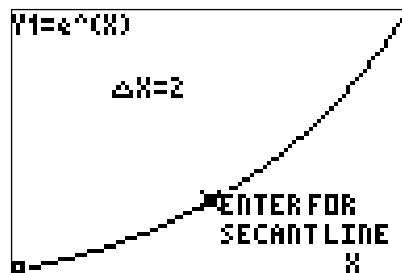
```

FINDS DIFFERENCE
QUOTIENT OF Y1
AT A SPECIFIED
POINT

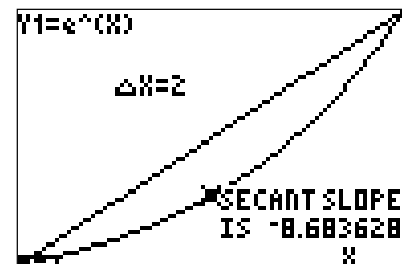
X VALUE: 1
INITIAL ΔX: 2

```

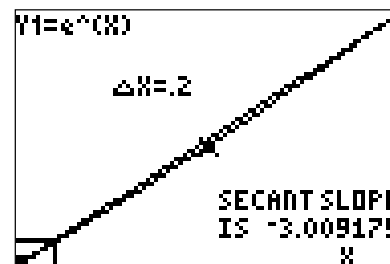
- Set  $X = 1$  and  $\Delta X = 2$ . The program will plot the function for  $1 < X < 1 + \Delta X$ .



- Press `ENTER` to see the secant line and its slope.



- Keep pressing **ENTER** to repeat the process, but with  $\Delta X$  reduced each time by a factor of 10. The box in the lower left-hand corner shows the area that will be graphed at the next step.
- When you've had enough, select QUIT from the menu.



As we ZOOM In more and more, the graph of  $f$  becomes straighter and straighter, so that it looks more and more like its tangent. The secant line, for which we can calculate the slope, approaches the tangent, so that the slope of the secant line is ultimately a good approximation to the slope of the tangent.

Try the program with other functions in  $Y_1$ .

*A convincing demonstration that even a nasty curve becomes straight if you magnify it enough (local linearity), provided the function is differentiable*

Plot the graph of  $\sin(20X)$  (Radian MODE), using a WINDOW of  $[0, 5, 1] \times [-1, 1.5, 0.5]$ . Put the cursor on one of the peaks and successively ZOOM In at the maximum. The sharp peak eventually looks like a (horizontal) straight line (the tangent to the curve). The function is differentiable there.

A contrasting function on which to try the same process is  $y = 1 - (x - 1)^{2/3}$ . ZOOM In successively at the peak. *Is this function differentiable at  $x = 1$ ?* What about  $y = |x|$  ( $\text{abs}(X)$ ) at  $x = 0$ ?

## 6 Discovering the Derivative by Exploration

Modified from: Judy Broadwin, *Discovering the derivative by exploration*, TI-82/83 Activities for Calculus, TI Web site.

We will put several different functions  $f(x)$  in  $Y_1$  and try to discover their derivatives using the difference quotient

$$DQ = \frac{f(x+h) - f(x)}{h},$$

where  $h$  is a small number. Store 0.0001 in memory H: 0.0001 **[STO]** **[ALPHA]** **[H]** **[ENTER]**.

Enter the difference quotient into  $Y_2$ :  $Y_2 = (Y_1(X+H) - Y_1(X))/H$ .

$Y_1$  is in the **[VARS]** Y-VARS Function menu. You can't just type **[Y]** **[1]**.

Turn off  $Y_2$ : move the cursor over the = sign and press **[ENTER]**.

**Exploration 1** *Discover the derivative of  $f(x) = e^x$*

Set  $Y_1 = e^x$ .

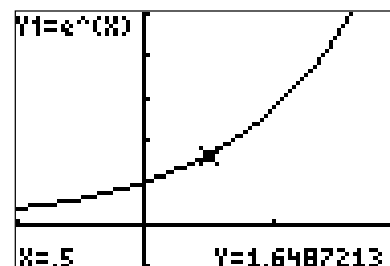
Set **[WINDOW]** to  $[-1, 2, 1] \times [-1, 5, 1]$ .

Press **[TRACE]**.

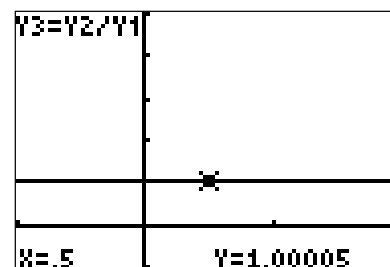
```

Plot1 Plot2 Plot3
\Y1=e^(X)
\Y2=(Y1(X+H)-Y1(X))/H
\Y3=Y2/Y1
\Y4=
\Y5=
\Y6=

```

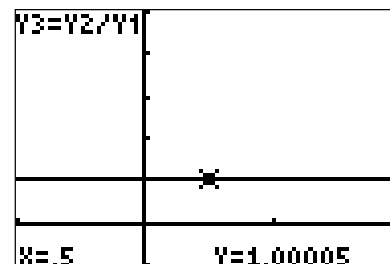


Then turn off  $Y_1$  and graph  $Y_2$ . *What do you see?*



Then set  $Y_3 = Y_2/Y_1$  and graph it. *What do you see?*

What is  $\frac{d}{dx}(e^x)$ ?



**Exploration 2** Discover the derivative of  $f(x) = \sin(x)$

Set  $Y_1 = \sin(X)$ .

Turn  $Y_1$  off and  $Y_2$  on.

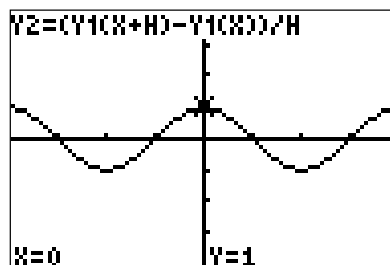
```

Plot1 Plot2 Plot3
\Y1=sin(X)
\Y2=(Y1(X+H)-Y1(X))/H
\Y3=cos(X)
\Y4=
\Y5=
\Y6=
    
```

Press **ZOOM** **7** and then **TRACE**.

Describe the graph of  $Y_2$ .

Do you recognise the function?



Can you describe the relationship between the graph of the function and the graph of its difference quotient?

Set  $Y_3 = \cos(X)$ .

Compare  $Y_2$  and  $Y_3$  using **TABLE**.

Why are they not identical?

X	Y2	Y3
0	1	1
1	.54026	.5403
2	-.4162	-.4161
3	-.99	-.99
4	-.6536	-.6536
5	.28371	.28366
6	.96018	.96017

**Exploration 3** Discover the derivative of  $f(x) = \ln(x)$

Set  $Y_1 = \ln(X)$  and turn it off. Leave  $Y_2$  on.

In **TBLSET**, set  $TblStart = 0$ ,  $\Delta Tbl = 1$ .

Press **TABLE**.

Can you guess the derivative of  $\ln(x)$ ?

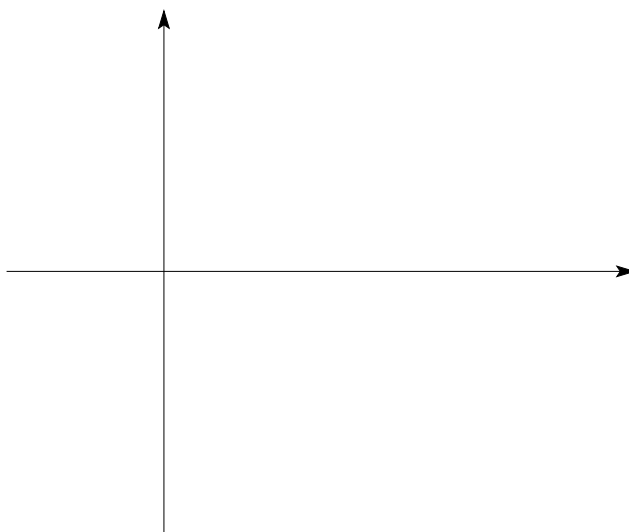
X	Y2	
0	ERROR	
1	.99995	
2	.49999	
3	.33333	
4	.25	
5	.2	
6	.16667	

## 7 Derivatives and the Shape of a Graph

From: Pat Forster, Alan Cadby and Gary Young, *This is I.T. Graphics-calculator activities for upper-secondary students* by Pat Forster, Alan Cadby and Gary Young, AAMT, 1998.

This investigation is to help you understand how the first and second derivatives of an equation can tell you the shape of its graph. You will also learn some new terminology to describe graphs.

- Copy from your calculator the graph of  $y = 5x^2 - 2x^5$  for  $-1 \leq x \leq 2$ ,  $-10 < y < 10$ . Put scales on your graph.



- Find and enter the equations for the first and second derivatives of  $y = 5x^2 - 2x^5$  into your calculator as Y2 and Y3. Look at the values of these derivatives for different values of  $x$  using the calculator's table of values as follows:

first press **TBLSET** and set  $TblStart = -1$ ;  $\Delta Tbl = 0.2$ ; then press **TABLE**.

- Use a red pen and put plus signs (+) along the sections of your graph above where  $dy/dx$  is positive. Similarly, put minus signs (-) and zero (0), where appropriate.
- Using a different-coloured pen, mark where  $d^2y/dx^2$  is +, - or zero. (Scroll up the table in the X column.)
- The points on a graph where  $dy/dx = 0$  are called **stationary points**. Fill in the table below.

Type of stationary point	Co-ordinates	$\frac{dy}{dx}$ (+, -, 0)	$\frac{d^2y}{dx^2}$ (+, -, 0)
<b>maximum</b>			
<b>minimum</b>			

3. (a) Sections of a graph where  $\frac{d^2y}{dx^2}$  is positive are described as **concave up**.

If  $\frac{d^2y}{dx^2}$  is negative, the curve is **concave down**.

For what values of  $x$  is your graph concave up? .....

For what values of  $x$  is your graph concave down? .....

Is the graph at the maximum point concave up or concave down? .....

Is the graph at the minimum point concave up or concave down? .....

(b) A point at which a curve is concave down ( $d^2y/dx^2$  negative) on one side and concave up ( $d^2y/dx^2$  positive) on the other side, *or vice versa*, is called a **point of inflection**. The sign of the second derivative must *change* as we pass through a point of inflection because the graph changes concavity.

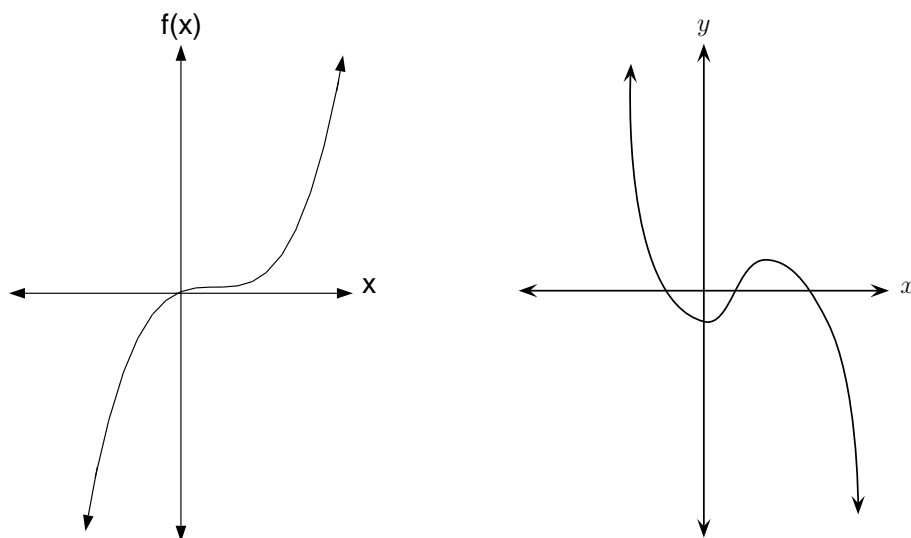
Use the table of values to find the point of inflection of the function here: change *TblStart* and  $\Delta Tbl$  to zoom in on the zero of  $Y_3$ . *Why do we look for a zero? Is this zero a point of inflection?* Write its co-ordinates on your graph.

(c) *Is  $x = 0$  a point of inflection of the function  $y = x^4$ ?* Give reasons.

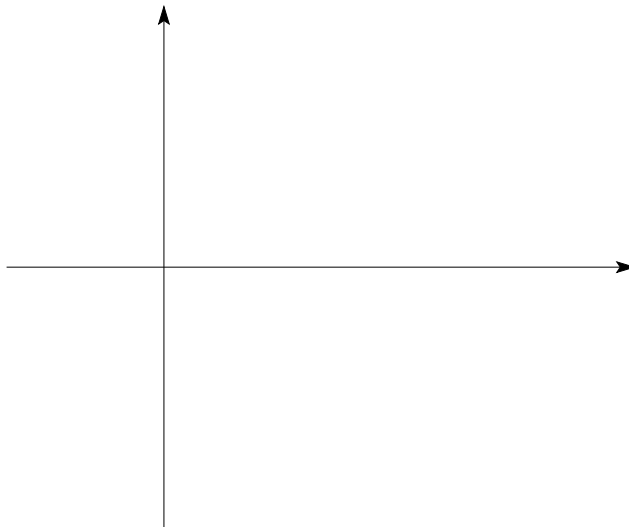
(d) Points of inflection can be the steepest section of a graph — look at the graph of  $y = \sqrt[3]{x-1}$  at  $x = 1$  (the cube root is MATH 4). They can also be stationary points (graph is horizontal) — look at the graph of  $y = x^3 + 1$  at  $x = 0$ . However, in general, the slope at a point of inflection can be any value.

Put a cross on the points of inflection in each of the following two graphs. *Is the point of inflection in either graph a stationary point?*

.....



4. Sketch a graph with the given properties.



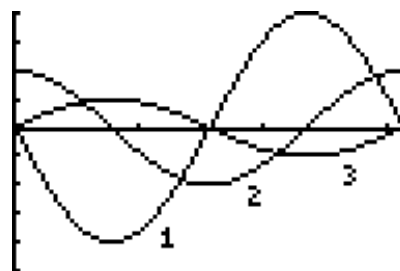
- (a) Endpoints  $(-2, 1)$  and  $(7, 6)$ .
  - (b) Stationary point at  $(1, -5)$  where  $\frac{d^2y}{dx^2}$  is positive.
  - (c) Stationary point at  $(3, 2)$  where  $\frac{d^2y}{dx^2} = 0$ .
  - (d)  $\frac{d^2y}{dx^2} = 0$  at  $(2, -1)$ ,  $\frac{d^2y}{dx^2}$  is negative between  $x = 2$  and  $x = 3$ , and  $\frac{d^2y}{dx^2}$  is positive for  $x > 3$ .
  - (e) Classify the points in (b) – (d) as a maximum, minimum or point of inflection.
- .....
- (f) For what values of  $x$  is the curve concave up?
- .....

## 8 Graphing Derivatives and Anti-derivatives

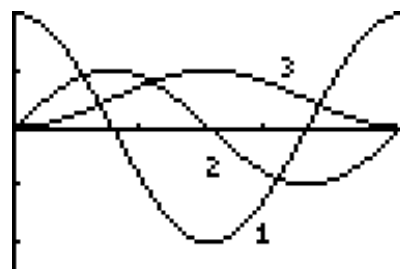
Based on: *How do you graph derivatives and anti-derivatives?* by John Maloney, Eightysomething! 7(1), 1997.

Here are two questions to challenge your students' understanding of the concepts of derivative and anti-derivative.

**Question 1:** The figure shows graphs of a function, its derivative and its second derivative. *Which curve is which?*



**Question 2:** The figure shows graphs of a function, its derivative and an anti-derivative. *Which curve is which?*



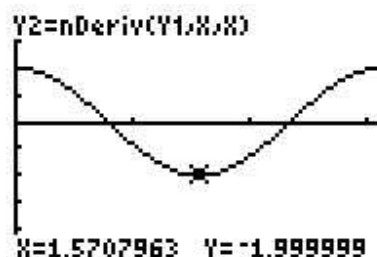
Let's investigate how to graph a function, its derivative and anti-derivative. Press **MODE** and make sure that all choices on the left are dark, so that in particular *Radian* and *Func* are selected.

Press **Y=**: set  $Y_1 = \sin(2X)$  and a WINDOW of  $[0, \pi, 1] \times [-5, 4, 1]$ . Press **TRACE**.

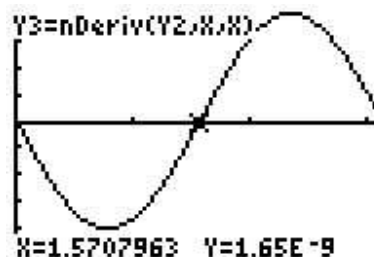


### Plotting derivatives

To graph the first derivative, set  $Y_2 = nDeriv(Y_1, X, X)$ . *nDeriv* is **MATH** **8**.  $Y_1$  is the function we are differentiating, the first  $X$  tells the calculator we wish to differentiate with respect to variable  $X$  and the second  $X$  is the value at which we wish to calculate the derivative — this value is set by the grapher as it plots successive points on the graph. Pressing **TRACE** gives you the figure.



Set  $Y_3 = nDeriv(Y_2, X, X)$  for the second derivative of  $Y_1$  — the derivative of the first derivative  $Y_2$ .



**Note:** The TI-83 will not allow you to calculate the third derivative in this manner — it would not be very accurate anyway. You need to use a different difference approximation<sup>6</sup> to the central or symmetric difference approximation used in *nDeriv*.

With  $Y_1$ ,  $Y_2$  and  $Y_3$  turned on, you should obtain the graph of the first question above. Students, of course, must use the relationship between a function and its derivative to answer the question.

```

Plot1 Plot2 Plot3
\Y1=sin(2X)
\Y2=nDeriv(Y1,X,
X)
\Y3=nDeriv(Y2,X,
X)
\Y4=fnInt(Y1(T),
T,0,X)

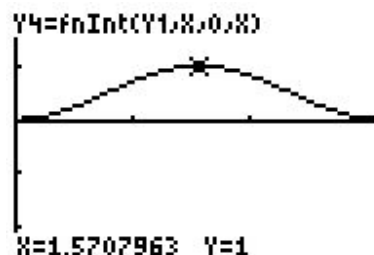
```

## Plotting an anti-derivative

To plot an anti-derivative of  $Y_1$ , set  $Y_4 = fnInt(Y_1(T), T, 0, X)$ . *fnInt* is  $\boxed{\text{MATH}} \boxed{9}$ .  $Y_1$  is the function we are integrating,<sup>7</sup>  $T$  is the dummy variable we are integrating with respect to, and 0 and  $X$  are the integration limits.

The function we are plotting is therefore  $y(x) = \int_0^x \sin(2t) dt$ .

Turn off  $Y_1 - Y_3$ , set  $Y_{min} = -2.5$ ,  $Y_{max} = 2$  and press  $\boxed{\text{GRAPH}}$ . Setting  $X_{res} = 3$  (plotting every third point) in  $\text{WINDOW}$  speeds up this graph without significant loss of resolution.  $\boxed{\text{ON}}$  stops graphing.



**Note:** The lower integration limit 0 is arbitrary. Change it to other numbers to show students that a function has (infinitely) many (related) anti-derivatives. The one graphed here is the one that passes through the origin.

With  $Y_1$ ,  $Y_2$  and  $Y_4$  turned on, you should obtain the graph of the second question above.

<sup>6</sup>  $f'''(x) \approx (f(x + 2\Delta x) - 2f(x + \Delta x) + 2f(x - \Delta x) - f(x - 2\Delta x))/2(\Delta x)^3$ .

<sup>7</sup> We could have put the function in directly as  $\sin(2T)$ . Similarly with the function in *nDeriv*. The advantage of the way we have set up the functions  $Y_2 - Y_4$  is that we can see the derivatives and anti-derivative of whatever function is in  $Y_1$ .

## The Chain Rule

Set

$$Y_5 = X^2$$

$$Y_6 = Y_1(Y_5)$$

$$Y_7 = \text{nDeriv}(Y_1, X, Y_5) * \text{nDeriv}(Y_5, X, X)$$

$$Y_8 = \text{nDeriv}(Y_6, X, X).$$

Plot and compare the graphs of  $Y_7$  and  $Y_8$  using a WINDOW of  $[-3, 3, 1] \times [-12, 12, 4]$ . Use **TRACE** and the up/down arrows to identify the two curves. Explain what you see.

## Answers to the Questions

### Question 1 *One possible answer*

Curve 3 is initially positive, so that it cannot be the derivative of either curve 1 or curve 2, both of which have negative slopes initially. Therefore, curve 3 must be the function.

The initial slope of curve 3 is positive, so that curve 1, which is initially negative, cannot be the first derivative of curve 3. Therefore, curve 2, which is initially positive, must be the first derivative of curve 3.

Curve 1, initially negative, must be the derivative of curve 2, and therefore the second derivative of curve 3.

### Question 2 *One possible answer*

Curve 3 cannot be the derivative of either curve 1 or curve 2 because it does not pass through zero when either of these curves has a maximum/minimum. Therefore, curve 3 must be either the function or an anti-derivative.

If curve 3 were the function, curve 2 could be its derivative (although its behaviour at either end doesn't look correct), but curve 1 could not be an anti-derivative because it is decreasing initially, whereas the anti-derivative of (positive) curve 3 must be increasing. Therefore, curve 2 must be the function and curve 3 an anti-derivative.

The anti-derivative of curve 2 must increase where curve 2 is positive and decrease where curve 2 is negative. Curve 3 has these properties. Because the area of curve 2 below the  $x$  axis is equal to the area above the  $x$  axis, the final  $y$  value of curve 3 must be equal to its initial  $y$  value.

Curve 1 must therefore be the derivative of curve 2. *Check:* when curve 2 has a maximum or minimum, curve 1 is zero; where curve 2 is increasing, curve 1 is positive, where curve 2 is decreasing, curve 1 is negative.

## 9 Rectangles, Area and the Definite Integral

The TI-83 program RIEMANN illustrates graphically how the area under a graph can be approximated by the areas of rectangles. As the number of rectangles covering the area increases, we obtain a better approximation to the area. Here we approximate  $\int_0^1 e^x dx$  by drawing rectangles (the sum of the areas is a *Riemann sum*).

- Put the function  $f(x) = e^x$  in Y1.
- Set a **WINDOW** of  $[0, 1, 0.2] \times [-0.3, 3, 1]$ . The value of  $-0.3$  for  $Ymin$  allows room at the bottom of the screen for displaying the cursor coordinates.
- Run the program: press **PRGM**, press the number against RIEMANN and press **ENTER** to start the program.
- Set the integration limits  $A = 0$  and  $B = 1$ .
- Set the number of rectangles  $N = 5$ .
- Choose the Left-Endpoint Rule (LER). The program will plot the function and draw in 5 rectangles, each rectangle touching the curve at its top-left corner. In this case, the area of the rectangles clearly underestimates the area under the graph.
- Press **ENTER** to see the area of the rectangles (LHSUM) as an approximation to the area under the graph.
- Press **ENTER**, set  $N = 5$  again and choose the Right-Endpoint Rule (RER). This time we obtain an overestimate of the area under the curve.
- Repeat the above steps, doubling the number of rectangles each time. Fill in the table below.<sup>8</sup> Your best estimate:  $\int_0^1 e^x dx \approx$  \_\_\_\_\_.

$N$	LER	RER	MEAN
5			
10			
20			
40			
80			

- When you've had enough, press **ON** Quit.

---

<sup>8</sup>The mean of the two estimates is equivalent to the Trapezoidal-Rule approximation to the area, a more accurate approximation for a given  $N$  than either the Left- or Right-Endpoint Rules.

**Answers**    *rounded to three decimal places.*

$N$	LER	RER	MEAN
5	1.552	1.896	1.724
10	1.664	1.806	1.720
20	1.676	1.762	1.719
40	1.697	1.740	1.718
80	1.708	1.729	1.718

Best estimate is 1.718.

The exact answer is  $e - 1 = 1.718$  to three decimal places.

## 10 Approximating Definite Integrals

Modified from an ADFA Lab, which is itself based on a lab in *Resources for Calculus, Volume 1: Learning by Discovery*, Anita Solow, editor, Mathematical Association of America Note 26, 1993.

In this lab, we shall be comparing several numerical approximations to

$$\int_0^1 (5x^4 - 3x^2 + 1) dx$$

with the exact answer obtained by algebraic integration. This will give us a feel for some of the methods of numerical integration, which we can then use for any function, including those which cannot be integrated algebraically.

### Question 1 *Algebraic integration — the exact answer*

What is the exact value of this integral? You may not realise it, but you are using the *Fundamental Theorem of Calculus* to do this definite integral exactly.

### Question 2 *Left-Endpoint and Right-Endpoint Rules*

One approach to numerical integration is to approximate the definite integral of  $y = f(x)$  with  $a \leq x \leq b$  by the areas of a number of rectangles under the curve. If a left-hand corner of each rectangle touches the curve, we have the *Left-Endpoint Rule*; if a right-hand corner of each rectangle touches the curve, we have the *Right-Endpoint Rule*. As the number of rectangles in the interval  $[a, b]$  gets larger and larger (covering the integration range  $a \leq x \leq b$  with more and more rectangles), both Rules give numbers closer and closer to the definite integral (exact answer).

- (a) On Figure 1 (at the end of this Lab), sketch and shade in the rectangles for the Left-Endpoint-Rule approximation to the definite integral  $\int_a^b f(x) dx$  with  $n = 4$  (four rectangles). *Note:* The function in Figure 1 is not the function in Question 1.
- (b) Using your sketch in (a), explain why the Left-Endpoint Rule with four rectangles approximates the area under the graph as

$$hf(x_0) + hf(x_1) + hf(x_2) + hf(x_3),$$

where  $x_0 = a$ ,  $x_4 = b$  and the width of each rectangle is  $h = (b - a)/4$ .

- (c) Use the RIEMANN program (instructions over) to estimate  $\int_0^1 (5x^4 - 3x^2 + 1) dx$  using the Left-Endpoint Rule with  $n = 4$ . A suitable WINDOW is  $[0, 1, 0.1] \times [-0.3, 3, 0.5]$ . Note that the integrand here is positive, so that the definite integral corresponds to the area under the graph of  $f$ .

- (d) Now use the RIEMANN program, doubling  $n$ , the number of rectangles, until two successive answers are the same when rounded to two decimal places. Write down the  $n$  value of the first of these two answers.

### Question 3 *The Trapezoidal Rule*

The Left-Endpoint and Right-Endpoint Rules approximate the area under a function by rectangles. In many cases, for example the function in Figure 1 with the rectangles you drew in, this is not a good approximation. We get a better approximation by using trapeziums: both top corners of each trapezium touch the curve.

- (a) On Figure 2, draw in and shade the trapeziums, the total area of which approximates the definite integral  $\int_a^b f(x) dx$ , again with  $n = 4$ .

The area of the trapezium in Figure 3 is  $h(r + s)/2$ . To see this result, split the trapezium into two regions — a triangle and a rectangle.

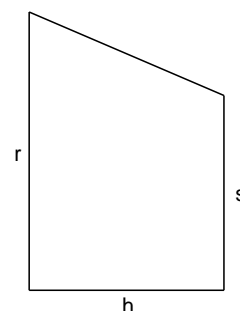


Figure 3

- (b) Using your sketch in (a) and Figure 3, explain why the Trapezoidal Rule with four trapeziums approximates the area under the graph as

$$hf(x_0) + 2hf(x_1) + 2hf(x_2) + hf(x_3),$$

where  $x_0 = a$ ,  $x_4 = b$  and the width of each trapezium is  $h = (b - a)/4$ .

- (c) Evaluate  $T_4$ , the Trapezoidal Rule with four trapeziums, as an estimate of the integral  $\int_0^1 (5x^4 - 3x^2 + 1) dx$  using the NUMINT program (T stands for Trapezoidal Rule). How does this result compare with the rectangle result and the exact answer?
- (d) Now use the NUMINT program, doubling  $n$ , the number of trapeziums, until two successive answers are the same when rounded to two decimal places. Write down the  $n$  value of the first of these two answers. Compare it with the rectangle  $n$  value.

**Question 4** *Simpson's Rule*

A picture of Simpson's Rule for which  $n = 4$  is given in Figure 4. We want to estimate the area under the solid curve. We do this by fitting parabolas to three successive points on the graph and adding up the areas under the parabolas.

The dashed line in Figure 4 shows two parabolas: one through  $(x_0, f(x_0))$ ,  $(x_1, f(x_1))$  and  $(x_2, f(x_2))$ ; the other through  $(x_2, f(x_2))$ ,  $(x_3, f(x_3))$  and  $(x_4, f(x_4))$ .

- (a) On Figure 4, shade the area calculated by Simpson's Rule as an approximation to the definite integral  $\int_a^b f(x) dx$ .
- (b) Evaluate  $S_4$ , Simpson's Rule with four divisions of the integration interval, as an estimate of  $\int_0^1 (5x^4 - 3x^2 + 1) dx$  (Simpson's Rule) using NUMINT. Note that the NUMINT version of Simpson's Rule uses  $2N$  sub-divisions, where  $N$  is the value you input ( $n$ , the number of divisions of the integration interval, must be even for Simpson's Rule). Compare your result with those from Questions 1 – 3.
- (c) Now use the NUMINT program, doubling the number  $n$  of divisions of the integration interval each time, until two successive answers are the same when rounded to two decimal places. Write down the  $n$  value of the first of these two answers. Compare it with the rectangle and trapezium values.

**Question 5** *Comparing the methods*

What conclusions can you draw from your results regarding the different methods for estimating the definite integral? Which method would you choose to use? Why?

**PROGRAMS**

These programs calculate approximate values for  $\int_A^B f(X) dX$ . The number  $N$  is an input to the program.

NUMINT approximates the integral using the *Left-Endpoint Rule* (**L**), the *Right-Endpoint Rule* (**R**), the *Trapezoidal Rule* (**T**) and the *Midpoint Rule* (**M**), **all with  $N$  sub-divisions**, and *Simpson's Rule* (**S**) **with  $2N$  sub-divisions** to ensure an even number of sub-divisions.

RIEMANN approximates the integral using the *Left-Endpoint Rule* or the *Right-Endpoint Rule* with  $N$  sub-divisions of the interval  $[A, B]$ , and draws the corresponding rectangles.

**Use:** Type the function to be integrated into Y1.  Quit stops both programs.

- For NUMINT, run the program and follow the prompts. Press  to input a different number  $N$  of sub-divisions.
- For RIEMANN, first set a suitable WINDOW to display the function (plot the function on the integration range first to check). Run the program and follow the prompts. Make sure  $B > A$ , otherwise things get mixed up. After the graph is plotted, press  to see the numerical approximation to the integral, and  again to do a new plot.

## Numerical Integration Lab Figures

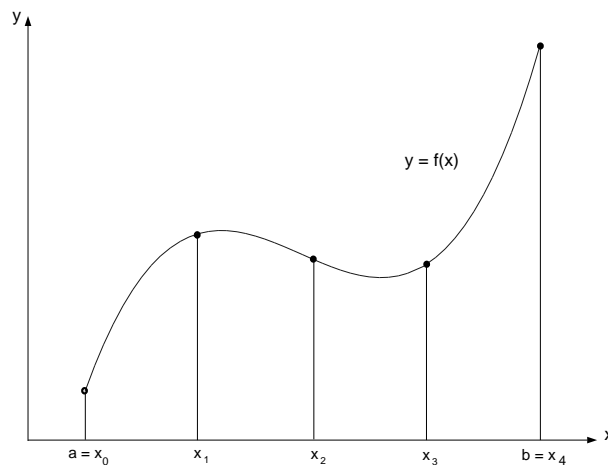


Figure 1: Left-Endpoint Rule

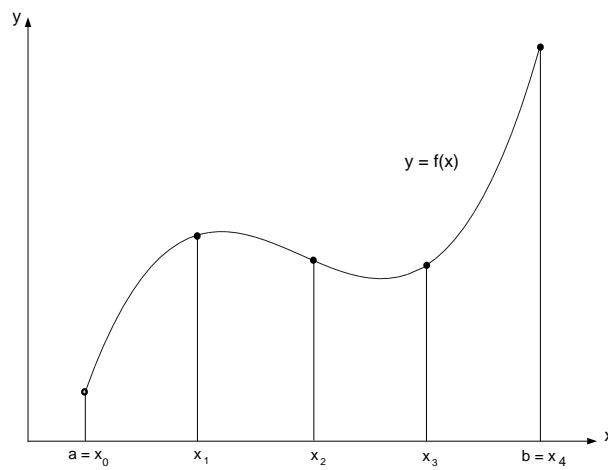


Figure 2: Trapezoidal Rule

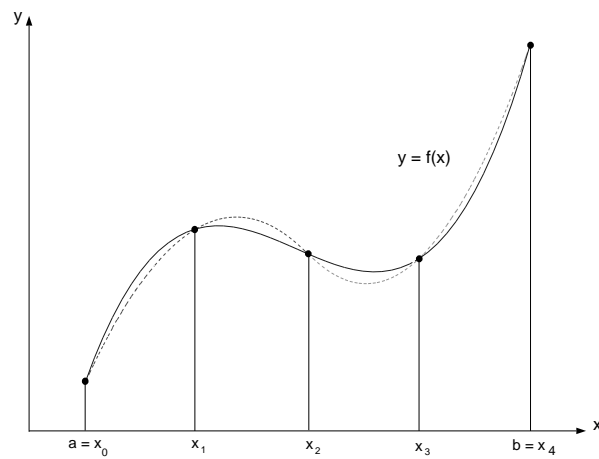


Figure 4: Simpson's Rule

## 11 Web Sites

- A good starting point for web links is the AAMT home page [www.aamt.edu.au](http://www.aamt.edu.au), in particular the links to other Maths sites under *Resources / Members' Sites / Mathematical Software and Technology*.
- Barry Kissane's home page [wwwstaff.murdoch.edu.au/~kissane](http://wwwstaff.murdoch.edu.au/~kissane) is another web site of general interest. This page contains some interesting discussion papers under Publications and lots of other useful stuff, including resources.
- [www.prenhall.com/divisions/esm/app/calculator](http://www.prenhall.com/divisions/esm/app/calculator) contains instructions on how to do most things with most models of graphics calculator. A terrific reference site for basic calculator operations.
- [education.ti.com](http://education.ti.com) — the TI site. Lots of stuff here.
- [otresources.ti.com](http://otresources.ti.com) The TI site for teaching material. Unfortunately, many of the resources now cost money, but there are some free sample activities.
- [education.ti.com/us/activity/books/overview.html](http://education.ti.com/us/activity/books/overview.html) takes you to the Explorations Series books; some of these books have sample chapters you can download.
- [education.ti.com/asiapacific/index.html](http://education.ti.com/asiapacific/index.html) is the local TI site. Especially useful if you need to contact TI in Australia about such things as calculator loan programs and TI PD sessions in Australia.
- [www.connecting-t3.com.au](http://www.connecting-t3.com.au) The local T<sup>3</sup> (Teachers Teaching with Technology) site. T<sup>3</sup> is the PD arm of TI.

### Web sites for programs

- The TI web site [education.ti.com/us/resources/programarchive/programarchive.html](http://education.ti.com/us/resources/programarchive/programarchive.html) contains an extensive list of programs for TI calculators which you can download into your calculator with TI CONNECT.
- [www.ticalc.org](http://www.ticalc.org) is another site that contains programs, as well as lots of other useful information about TI calculators (but no teaching resources).

The quality of programs varies enormously on both these sites, but you find plenty of ideas of what is possible, even if the programs don't work too well. They can always be fixed up.

- At our ADFA website

[www.unsw.adfa.edu.au/pems/news/high\\_school/hsc\\_activities.html](http://www.unsw.adfa.edu.au/pems/news/high_school/hsc_activities.html),

we have a number of TI-83 programs (with instructions) used in our Calculus, Linear Algebra and Probability courses.