Complex Numbers on a TI-83/83+

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At *www.unsw.adfa.edu.au/pems/news/high_school/hsc_activities.html*

- A variety of graphics-calculator activities for Years 9 and 10 — written as part of the CQTP Program.
- TI-83 programs and program information.
- *Using the TI-83/83+* — an introduction to the basic operations, suitable for Years 8–12.
- *The Graphics Screen and Accuracy* — information to help you understand the graphical and numerical limitations of a graphics calculator.
- *Coordinate Geometry on a TI-83/83+* — basic commands and a variety of problems, suitable for Years 9 and 10.
- *Population Modelling* — a variety of problems from simple exponential growth to Leslie matrices and difference equations, covering Years 7–12.
- *Calculus on a TI-83/83+* — basic commands and a variety of problems, suitable for Years 11 and 12.
- *Matrices on a TI-83/83+*— basic commands and a variety of problems, suitable for Years 11 and 12.
- *Sequences and Series on a TI-83/83+*— basic commands and a variety of problems, suitable for Years 11 and 12.
- *Introduction to Complex Numbers* — complex numbers from the beginning, covering the basic operations, but set in the context of complex numbers as a mathematical structure.
- *Programming a TI-83/83+* — suitable for teachers and keen students.

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1 Setting Complex Mode

Press [MODE] and select the Cartesian form $a + bi$ (2nd line from bottom) with the cursor and [ENTER]. Press [2nd] [QUIT] to return to the home screen.

We shall use $z_1 = 1 + 2i$ and $z_2 = 3 - i$ in our examples. i is [2nd] [·].

Complex numbers can be stored in the same way as ordinary numbers. Store z_1 in memory A: $1 + 2i$ [STO] A; z_2 in memory B: $3 - i$ [STO] B.

2 Basic Operations

2.1 Addition and subtraction

Just as you would expect.

$1 + 2i + 3 - i = 4 + i$ or $A + B = 4 + i$

$1 + 2i - (3 - i) = -2 + 3i$ or $A - B = -2 + 3i$

2.2 Multiplication and division

Again as you would expect. Implied multplication works too.

$(1 + 2i)(3 - i) = 5 + 5i$ or $AB = 5 + 5i$

$(1 + 2i)/(3 - i) = 0.1 + 0.7i$ or $A/B = 0.1 + 0.7i$

2.3 Conjugation

Finding the complex conjugate.

$\bar{z}_1 = \text{conj}(1 + 2i) = 1 - 2i$ conj is [MATH] CPX [1]

2.4 Real part

$\text{Re}(z_1) = \text{real}(1 + 2i) = 1$ real is [MATH] CPX [2]

2.5 Imaginary part

$\text{Im}(z_1) = \text{imag}(1 + 2i) = 2$ imag is [MATH] CPX [3]

2.6 Modulus

Sometimes called length or absolute value.

$|z_1| = \text{abs}(1 + 2i) = \sqrt{1^2 + 2^2} = \sqrt{5} \approx 2.236$ abs is [MATH] CPX [5]

3 Polar Form

The polar form (r, θ) of a complex number is represented on the TI-83 in exponential form $re^{i\theta}$, where r is the modulus or length and θ is the angle. This form is equivalent to the form $r\big(\cos(\theta) + i\sin(\theta)\big)$, often abbreviated as $r\text{cis}(\theta)$.

3.1 Modulus and angle

Use the MATH CPX operations *abs* and *angle* to extract out the modulus and angle respectively of a complex number in either Cartesian or polar form.[1]

Similarly, *real* and *imag* can be used to find the real and imaginary parts respectively of a number in polar form, and *conj* its complex conjugate.

3.2 Form of output

Complex numbers can be input in either Cartesian or polar form, no matter what the MODE selection, but the output form will depend on whether $a + bi$ (Cartesian) or $re^{i\theta}$ (polar) is selected in MODE.

When you are inputting in polar form, it is VITAL that you are in Radian MODE (see the Appendix), even if you wish to input an angle θ in degrees.

3.2.1 Cartesian ($a + bi$) MODE

Input $\sqrt{2}e^{i\pi/4}$ using the e^x key and press ENTER.

$\sqrt{2}e^{i\pi/4} \quad \rightarrow \quad 1 + i.$

Try the famous calculation $e^{i\pi} + 1$.

To input an angle in degrees, use the degree symbol ANGLE 1.

$\sqrt{2}e^{i45^\circ} \quad \rightarrow \quad 1 + i.$

3.2.2 Polar ($re^{i\theta}$) MODE

Set Polar using MODE. Make sure you are still in Radian MODE. Enter the number $1 + i$ (Cartesian form) and press ENTER.

$1 + i \quad \rightarrow \quad 1.414213562e^{.7853981634i}$ use the right arrow to scroll across

If you repeat this in Degree MODE,

$1 + i \quad \rightarrow \quad 1.414213562e^{45i}$,

[1]Using the program AAANSWER (see the Appendix) on these numbers is often useful.

i.e. the angle is displayed in degrees. However, if you do not change back to Radian MODE to input polar form, any input with an angle in degrees will be wrong (see the Appendix).

A better alternative to changing back and forth between Degree and Radian MODE when you are using angles in degrees is to stay in Radian MODE and use the program CMPXANGD to convert your output to degrees.

$1 + i \quad \rightarrow \quad 1.414213562e^{\cdot 7853981634i}$.

Run CMPXANGD to give

LENGTH (modulus) = 1.414213562 (stored in memory/variable R)

ANGLE(°) = 45 (stored in θ)

The CMPXANGD program is listed in the Appendix.

3.3 Form of input

In carrying out calculations using complex numbers, you can input the numbers in any form, Cartesian or polar or mixed. The MODE setting only determines the output form of the answer.

$$1 + i + \sqrt{2}e^{i45^\circ} = 2.828427125e^{\cdot 7853981634i} \quad \text{Polar form}$$

$$= 2 + 2i \quad \text{Cartesian form.}$$

3.4 Conversion between forms

To convert complex numbers from one form to the other, use the conversions in the [MATH] CPX menu.

$1 + i$ [▶Polar] [ENTER] gives $1.414213562e^{\cdot 7853981634i}$.

$\sqrt{2}e^{i\pi/4}$ [▶Rect] [ENTER] gives $1 + 1i$.

4 Powers and Roots

4.1 Powers

Integer powers work as you would expect.

$(1+2i)^4 = -7 - 24i.$

$\left(\sqrt{2}e^{i\pi/4}\right)^4 = 4e^{i\pi} = -4.$

4.2 Roots

Unfortunately you only get one root of a complex number when you use square root, cube root, xth root and fractional powers. To find all the roots,[2] you can use the program CMPXROOT (see the Appendix), which calculates and plots the nth roots of a given complex number.

5 Exercises

$$z_1 = 3 + 4i \qquad z_2 = 2 + 3i \qquad z_3 = \sqrt{2}e^{i\pi/4} \qquad z_4 = e^{i\pi/2}$$

Find

1. $z_1 + z_2$
2. $2z_1 + 3z_2$
3. $z_1 - z_2$
4. $4z_1 - 2z_2$
5. $z_1 z_2$
6. z_1/z_2
7. $\bar{z}_1$
8. $z_1\bar{z}_1$
9. $|z_1|^2$
10. $\mathrm{Re}(z_1)$
11. $\mathrm{Im}(z_2)$
12. z_1^2
13. z_1^4
14. $\sqrt{z_1}$
15. $z_3 z_4$ in polar form
16. z_3/z_4 in polar form
17. $\sqrt{z_4}$ in polar and Cartesian form
18. z_1 in polar form
19. z_3 in Cartesian form
20. z_4 in Cartesian form

[2] A complex number has n nth roots (n an integer).

6 Appendix

6.1 Input/Output of Complex Numbers

MODE		INPUT	OUTPUT	CORRECT?
$a+bi$	Radian	$\sqrt{2}e^{i\pi/4}$	$1+1i$	YES
		$\sqrt{2}e^{i45^\circ}$	$1+1i$	YES
	Degree	$\sqrt{2}e^{i45}$	$0.7429+1.203i$	NO
		$\sqrt{2}e^{i45^\circ}$	$0.7429+1.203i$	NO
$re^{i\theta}$	Radian	$1+i$	$\sqrt{2}e^{i\pi/4}$	YES
		$\sqrt{2}e^{i45^\circ}$	$\sqrt{2}e^{i\pi/4}$	YES
	Degree	$1+i$	$\sqrt{2}e^{i45}$	YES
		$\sqrt{2}e^{i45^\circ}$	$\sqrt{2}e^{58.31i}$	NO
		$\sqrt{2}e^{i45}$	$\sqrt{2}e^{58.31i}$	NO

To input, always use Radian MODE. Use the degree symbol (ANGLE 1) if the angle is in degrees.

6.2 Programs

All the programs listed in these notes can be found at the web site

www.unsw.adfa.edu.au/pems/news/high_school/hsc_activities.html

You will need *TI Connect*, available free at

education.ti.com/us/product/accessory/connectivity/features/software.html

(click on *download* at left to download) and a special cable (costs money) to download programs from your computer to your calculator.

6.2.1 AAANSWER

Converts an answer to surds or multiples of π. Run this program when you have a decimal answer that might be a square or cube root or a multiple of π. It will tell you. A very useful program taken from the web (author unknown).

6.2.2 CMPXANGD

Displays the modulus R and angle θ (in degrees) of a complex number. Run the program once you have the complex number as a result. The arrow → in the program below represents the STO key.

Ans → Z

abs(Z) → R

180 angle(Z)/π → θ

Disp "MODULUS, ANGLE(∘)",R, θ

Output(8,1," ")

6.2.3 CMPXROOT

Displays the rectangular values (x, y) and plots the Nth roots of the complex number $A + iB$, i.e. the points $Z = (A + iB)^{\frac{1}{N}}$, on the Argand diagram. The roots all lie on a circle which is drawn. Lines are drawn from the origin to each root to show where the root is and to make the symmetry of the roots more obvious. Note that if a root Z lies on a co-ordinate axis, you won't see it, but it should be obvious from symmetry.

Use: Run the program. Input values for A, B and N (positive integer) when prompted. The program displays each root and then plots it. Press [ENTER] to continue at each step. The roots are also stored in list L_6 if you need them later. Move the cursor around the final plot using the arrow keys to see rectangular (x, y) values if [FORMAT] is set to RectGC or polar (r, θ) values if [FORMAT] is set to PolarGC.

Example: The cube roots (N = 3) of $1 + 2i$ are $1.220 + 0.472i$, $-1.018 + 0.820i$ and $-0.201 - 1.292i$, all rounded to three decimal places.

7 Answers to Exercises

1. $5 + 7i$
2. $12 + 17i$
3. $1 + i$
4. $8 + 10i$
5. $-6 + 17i$
6. $18/13 - i/13$
   Frac (MATH 1) is useful here
7. $3 - 4i$
8. 25
9. 25
10. 3
11. 3
12. $-7 + 24i$
13. $-527 - 336i$
14. $2 + i$ and $-2 - i$
15. $\sqrt{2}e^{i3\pi/4}$
    Use *abs*, *angle* and AAANSWER
16. $\sqrt{2}e^{-i\pi/4}$
17. $\pm e^{i\pi/4}$ or $\pm(1 + i)/\sqrt{2}$
18. $5e^{0.927295218i}$
19. $1 + i$
20. i