Coordinate Geometry on a TI-83/83+

Peter McIntyre

Contents

Contact for more help

Peter McIntyre

School of Physical, Environmental
and Mathematical Sciences
UNSW@ADFA
Canberra ACT 2600

Email: p.mcintyre@adfa.edu.au
Phone: (02) 6268 8896
FAX: (02) 6268 8786

At *www.unsw.adfa.edu.au/pems/news/high_school/hsc_activities.html*

- A variety of graphics-calculator activities for Years 9 and 10 — written as part of the CQTP Program.
- TI-83 programs and program information.
- *Using the TI-83/83+* — an introduction to the basic operations, suitable for Years 8–12.
- *The Graphics Screen and Accuracy* — information to help you understand the graphical and numerical limitations of a graphics calculator.
- *Population Modelling* — a variety of problems from simple exponential growth to Leslie matrices and difference equations, covering Years 7–12.
- *Calculus on a TI-83/83+* — basic commands and a variety of problems, suitable for Years 11 and 12.
- *Sequences and Series on an TI-83/83+* — basic commands and a variety of problems, suitable for Years 10–12.
- *Matrices on a TI-83/83+* — suitable for Years 11 and 12.
- *Complex Numbers on a TI-83/83+* — suitable for Years 11 and 12.
- *Introduction to Complex Numbers* — complex numbers from the beginning, covering the basic operations, but set in the context of complex numbers as a mathematical structure.
- *Programming a TI-83/83+* — suitable for teachers and keen students.

All the programs listed in these notes can be found at the above web site. You will need *TI Connect*, available free at

education.ti.com/us/product/accessory/connectivity/features/software.html

(click on *download* at left to download) and a special cable (costs money) to download programs from your computer to your calculator.

1 Terminology and Useful Keys

- **Home screen:** the screen where you type in calculations and commands. Press QUIT (2nd MODE) to return to the home screen.
- **CLEAR:** CLEAR clears the line you are currently typing. If the current line is already clear and the cursor is on a blank line, CLEAR clears the whole screen. In a menu, CLEAR exits the menu if you don't want to select a menu item.
- **DEL INS:** DEL deletes the character the cursor is on. INS allows you to insert characters before the character the cursor is on. To turn INS off, press INS again or just move the cursor with the arrow keys.

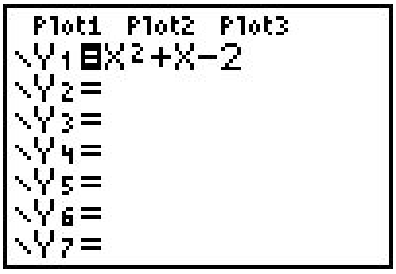
2 Basic Operations

2.1 Graph $f(x) = x^2 + x - 2$ for $-3 < x < 3$

- Press **Y=**: set $Y_1 = X^2 + X - 2$. The independent-variable key X, T, θ, n gives X.

  Note the highlighted = sign, which means the function will be plotted when you press GRAPH.

  Move the cursor over the = sign and press ENTER to toggle the function off/on.

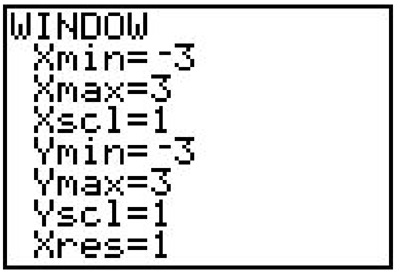


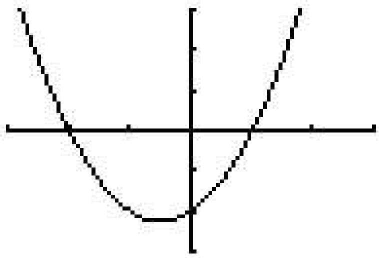
- Press **WINDOW**: specify the viewing window as shown in the figure.

  Note the difference between the blue − key (subtract) and the white (−) key (change sign).

  Use ENTER to move between values.

  Xscl, Yscl are the distances between tick marks on the axes (0 gives no tick marks).





- Press **GRAPH**: graph the function.

2.2 Generating a table of function values

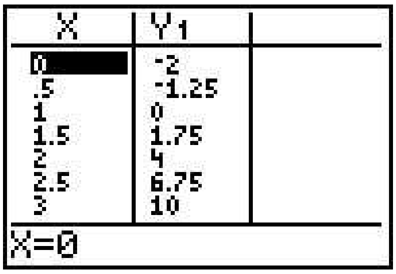
- Press [Y=] and make sure the = signs of the functions you want are highlighted.

- Set the table 'WINDOW' using [TBLSET]:
  TblStart = 0, ΔTbl = 0.5, Auto Auto.

  (Select with the cursor and [ENTER].)
  This generates values automatically, starting at X = 0 and incrementing in steps of 0.5.

- Press [TABLE] (on the [GRAPH] key); *scroll up and down in the X column, down in the Y column(s).* Note full display of highlighted number at the bottom of the screen.

  Scroll to the function header to see the formula. You can actually change the formula here by pressing [ENTER], editing it and pressing [ENTER] again.



- To enter your own X values, set *Indpnt* to *Ask* in [TBLSET] and press [TABLE].

2.3 Find (estimate) $f(0.5)$

- **From a table** — see above.

- **On the graph**

  - Press [TRACE] (*top row of keys*).
  - Type in the X value, [.] [5] [ENTER], to move to the desired point on the graph. Note the coordinates at the bottom of the screen.
  - Alternatively, use the left and right arrow keys to move the cursor along the curve (but note the problem that arises when trying to reach X=0.5). The up and down arrows move between functions if there is more than one graphed.

- **On the home screen**

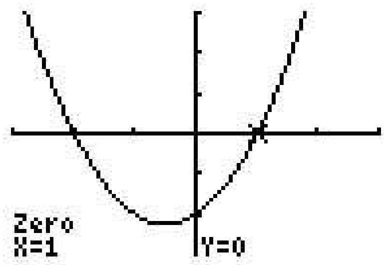
  - The calculator knows f by the name Y1. We need to evaluate Y1 (0.5).
    Y1 is in the [VARS] Y-VARS Function menu: [VARS] [▷] [1] [1]. You can't just type [Y] [1]. Follow with standard function notation (0.5) and [ENTER].

- *Answer*: $f(0.5) = -1.25$.

2.4 Find the zeros of $f(x) = x^2 + x - 2$

The graphical method in this and all the following operations is usually more meaningful than the home-screen operation, which is really just a 'black box'. Home-screen operations are in the Appendix.

- For a rough estimate, press TRACE and move the cursor as close as possible to the zero. Read the cursor coordinates at the bottom of the screen. Watch the Y coordinate to see when it changes sign. Zooming in on the zero — ZOOM 2, move the cursor to the zero, press ENTER — will produce greater accuracy with this method.
- For a more accurate estimate, use *zero* in the CALC menu (on TRACE).[1]
  The calculator asks for a left bound: move the cursor somewhere to the left of the zero you want (or type in a value) and press ENTER. For the right bound, move the cursor somewhere to the right of the zero you want (or type in a value) and press ENTER. Similarly for *Guess*. The guess doesn't have to be spot on. The calculator will then approximate the zero. Your bounds and guess will determine which zero you find if there are several.

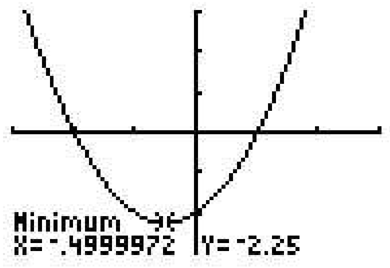


- *Answer*: $f(x) = 0$ when $x = -2, 1$.

2.5 Find the minimum of $f(x) = x^2 + x - 2$

- For a rough estimate, press TRACE and move the cursor as close as possible to the minimum. Read the cursor coordinates at the bottom of the screen. Watch the Y coordinate. Zooming in on the zero — ZOOM 2, move the cursor to the zero, press ENTER — will produce greater accuracy with this method.
- For a more accurate estimate, use *minimum* in the CALC menu. *minimum* works the same way as *zero* above. Use the bounds and the guess to pick out the minimum you want if there is more than one. Claimed accuracy is 10^{-5}.
- Finding maxima using the *maximum* command works in exactly the same way.

[1] If you have more than one curve, select the one you want with the up/down arrow keys before the next step.

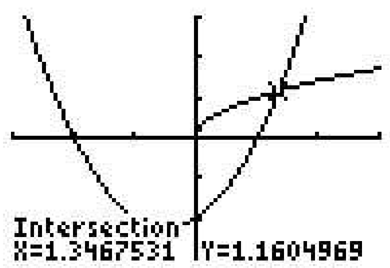


- *Answer*: the minimum value of $y = -2.25$ occurs at $x = -0.5$, rounded to 5 decimal places (the claimed accuracy).

2.6 Solve $x^2 + x - 2 = \sqrt{x}$

- Graph Y1 = X^2 + X − 2 and Y2 = $\sqrt{}$(X) using a suitable WINDOW.
- For a rough estimate, press TRACE and move the cursor as close as possible to the intersection. Read the cursor coordinates at the bottom of the screen. Zooming in on the zero — ZOOM 2, move the cursor to the intersection, press ENTER — will produce greater accuracy with this method.
- For a more accurate estimate, use *intersect* in the CALC menu.

  Press ENTER to select each curve.[2] Move the cursor to provide the *guess* (or type in an X value) and press ENTER. Your *guess* will determine which solution you find if there are several.



- *Answer*: $x^2 + x - 2 = \sqrt{x}$ when $x \approx 1.347$, $y \approx 1.160$. Answers rounded to 3 decimal places.

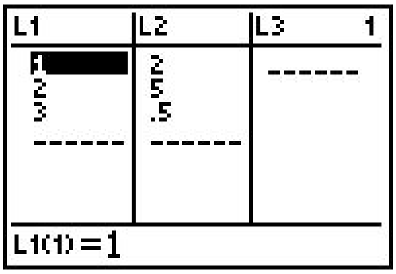
[2] The cursor starts on the first curve (usually Y1). The cursor automatically moves to the second curve after you press ENTER the first time. If you have more than two curves, select the curves you want with the up or down arrow keys (look at the top left of the screen) before pressing ENTER.

2.7 Graphing data points and points joined by lines

There are three possible ways to do this.

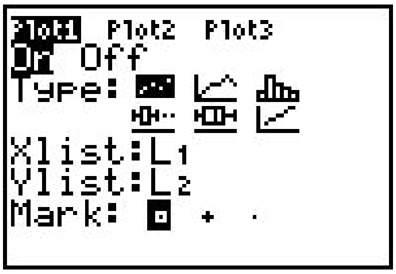
- **Series of points:** First you need to put the x values of the points in one list, the y values in another. The standard lists on the TI-83 are L1 – L2 (2nd functions on the corresponding number keys), but you can also give lists names if you want.

  The easiest way to enter the data is to press STAT and select Edit. . . .[a] Enter the x values in L1, pressing ENTER after each value including the last. Similarly, put the y values in L2.

  Now you need to tell the calculator what type of plot we want, where the data are and the type of marker for each point. Press 2nd Y= to access the STATPLOT menu. Up to three plots can be displayed. Press 1 to select Plot1.

  By moving the cursor and pressing ENTER, select the following options in the menu:

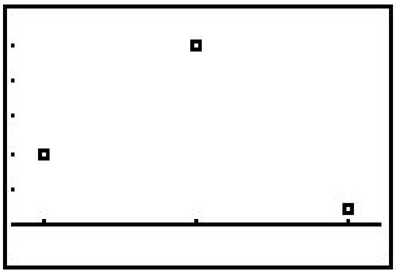
  

  On — turn the plot on

  the type of plot — the first is a scatterplot (just the points), the second a line plot (successive points joined by a line)

  specify the lists (probably already set to L1 and L2); and

  select the marker you want.

  Now press Y=. You should see Plot1 highlighted, indicating it will be plotted when you press GRAPH. Turn off any functions turned on, unless you want them plotted too. You can turn plots on and off here in the same way.

  Next the WINDOW. Either enter suitable values using WINDOW and then press GRAPH, or use ZOOM 9 (ZoomStat), which automatically sets a window and plots the graph.

[a] If you don't see columns for L1, L2 and L3, press STAT 5 ENTER to reset the list editor. To clear a list, move the cursor to the list header and press CLEAR.

- **Line segment:** use the *Line* command in the DRAW menu ([PRGM] key).

  From the home screen: Line (X1, Y1, X2, Y2). *Example*: Line (3, 4, 5, 2) draws the line (segment) between (3, 4) and (5, 2).[3]

  On a graph: select *Line*, move the cursor to one end of the line segment you want and press [ENTER]. Move the cursor to the other end of the line and press [ENTER] again. Keep drawing as many line segments as you want using [ENTER] to begin and end each segment. Press [CLEAR] to cancel the *Line* command.

- **Point:** use the *Pt–On* command in the DRAW menu.

  From the home screen: Pt–On (X, Y). *Example*: Pt–On (3, 4) plots the point (3, 4).

  On a graph: select *Pt–On*, move the cursor to the point(s) you want and press [ENTER]. Press [CLEAR] to cancel the *Pt–On* command.

2.8 Shading regions

There are eight possible graph styles, selected by moving the cursor to the left of the Y in the function definition (in [Y=]). Press [ENTER] successively to toggle through the various possibilities. Try graphing a function with each type. Shading can be useful for inequalities or linear programming.

- standard line
- heavy line
- shade above the function
- shade below the function
- moving ball with trail
- moving ball without trail
- dotted line.

For more intricate shading, use the *Shade* command in the DRAW menu. See the manual for details.

[3] To erase a line segment, use the *Line* command with a fifth argument 0: Line (3, 4, 5, 2, 0) erases the above line.

2.9 Appendix: Home-screen operations

Finding zeros: Use the interactive *Solver* (MATH 0).

Move the cursor to the top line if necessary, enter the equation as 0=X²+X−2 and press ENTER.

Set the bounds to $\{-3, 3\}$ (a list with curly brackets), enter a guess for X (this will determine which of the two possible values you find) and with the cursor anywhere on the X line, press ALPHA ENTER for SOLVE.

Scroll along the answer if there are dots after the last digit.

Note that this is an approximate numerical method, so that the answer is probably not exact.

Answer: $f(x) = 0$ when $x = -2$, 1.

You can also use the *solve* command, found in the CATALOG.

Finding minima: *fMin(Y1, X, –3, 3)* in the MATH menu: X is the independent variable in Y1; the last two inputs are the bounds. There is an optional fifth argument, the tolerance: the default is 10^{-5}.

Answer: the minimum value of $y = -2.25$ occurs at $x = -0.5$, rounded to 5 decimal places.

Finding maxima using the *fMax* command works in exactly the same way.

Solving equations: Write the equation as $0 = x^2 + x - 2 - \sqrt{x}$ and use the interactive *Solver*. See *Finding zeros* above.

Answer: $x^2 + x - 2 = \sqrt{x}$ when $x \approx 1.347$, $y \approx 1.160$. Answers rounded to 3 decimal places.

3 Activities

3.1 Linear Models

Renting a car

The Rent-a-Wreck Car Rental Company has the cheapest car rentals in town.

You can choose one of two options.

- **Option A** — no flat fee, but a charge of 28c per kilometre.
- **Option B** — a flat rate of $36 per day, plus 18c per kilometre.

(a) We want to hire a car for one day.

Under Option A, how much will it cost in dollars to drive 1 kilometre? 2 kilometres? 10 kilometres? 100 kilometres? x kilometres?

For Option A, what is the equation for the cost in dollars y in terms of the number of kilometres driven x? Check your equation with the numbers you calculated above.

Under Option B, how much will it cost in dollars to drive 1 kilometre? 2 kilometres? 10 kilometres? 100 kilometres? x kilometres?

For Option B, what is the equation for the cost in dollars y in terms of the number of kilometres driven x? Check your equation with the numbers you calculated above.

(b) Graph the equations for the two options, assuming we will drive up to 600 kilometres in a day.

(c) Estimate from the graph how far we have to drive before Option B becomes cheaper.

(d) From the graph, what is the slope of the line for Option A?
Hint: Press [TRACE] and use the left/right arrow keys to find two points on the graph; use these work out the slope.

What is the slope of the line for Option B?

What does the slope represent in this problem?

(e) What is the y intercept of the graph of Option A?
Hint: Press [TRACE], type in 0 and press [ENTER].

What is the y intercept of the graph of Option B?

What does the y intercept represent in this problem?

(f) Work out *exactly* how far we have to drive before Option B becomes cheaper.

Marketing a computer game

You have just written a cool computer game and your company wants to sell it. *How much should it charge?*

If it puts on a high price, the company won't sell as many games, but it will make more money per game sold. If the game is sold for a low price, the company won't make as much money on each game sold, but it will sell more games.

Clearly, the number sold depends on the price. Economists often assume that the number sold and price form a *linear equation.*

After doing some market research, the company thinks that if it sells the game for $160 per copy, it will sell about 800 copies. If the price is dropped to $40 per copy, it should sell about 8000 copies.

(a) Let's use a graph of number sold versus price to help us in our problem. Price will be on the x axis and number sold on the y axis. What are suitable scales for the two axes?

(b) What are the two points that we know on the graph of number sold versus price? Use the Line command in the DRAW menu to draw a straight line between these two points. The syntax is (from the home screen) Line (x_1, y_1, x_2, y_2), where (x_1, y_1) and (x_2, y_2) are the two points.

(c) What is the slope of the line? *Hint:* Use two points on the graph to work out the slope.

(d) What is the equation of the line? What extra information about the line do we use here? Graph the line and check that the points you know actually lie on the graph of the line.

(e) Does this graph tell us the answer to the question of what the price should be?

(f) Revenue means total income. It is the product of price and number sold. Write down the equation for revenue as a function of price x. What kind of function is this?

(g) Plot the graph of revenue versus price. Estimate the revenue if the price is $50.

(h) What is the best price to sell the game at? Why is it best? What is the revenue?

(i) What is the revenue if the game is sold at a price of $180? Explain.

Acknowledgement to material from an unknown website.

Linear Models Assignment

Brock has $_____ in the bank, has no income, but is spending about $_____ a week on his new girl friend Amber.

His sister Minerva has only $_____ in the bank, but is spending nothing and saving about $_____ a week.

(a) Find the equation of the line that gives how much money each person has in the bank as a function of time in weeks.

(b) What is the slope of each line?

(c) What is the physical interpretation of the slope?

(d) What is the y intercept of each line?

(e) What is the physical interpretation of the y intercept?

(f) When will Minerva and Brock have the same amount of money?

(g) When will Minerva have twice as much money as Brock?

The following Coordinate Geometry activities are available separately at

www.unsw.adfa.edu.au/pems/news/high_school/hsc_activities.html

Solutions and teacher's notes are also provided.

3.2 Coordinate Geometry Art

A simple picture consisting of straight-line segments is 'coded' using the coordinates of the points of its vertices. These are used 'transmit' the picture to someone else. A graphics calculators is used to 'decode' and check the 'transmitted' picture.

Year 9, Levels 1 and 2; Algebra; Coordinate Geometry.

3.3 A Classic Problem — The Hare and Tortoise

The graphs of the distances covered versus time in this classic race are used to answer various questions about the race, such as who won and by how much. A fun exercise in putting questions into maths and solving equations graphically.

Year 10, Level 1; Algebra; Sketching Other Graphs, Simultaneous Equations.

3.4 The Best Shape for a Can

Minimising the surface area of a cylinder (can) for a fixed volume. Numerical and graphical techniques, rather than Calculus, are used to find the minimum. Aspects of mathematical modelling are introduced.

Year 10, Level 1; Algebra/Measurement; Sketching Other Graphs/Volume.

3.5 Parabolic Aerobics

The first activity investigates the effect of changing the numbers A, B and C on the graphs of the family of parabolas $Y=A(X-B)^2+C$. In the second activity, you have to guess the numbers A, B and C for the graph of a mystery parabola generated by the PARABOLA program. The calculator checks your answers and keeps score.

Year 9, Levels 1 and 2; Algebra; Coordinate Geometry.

3.6 Alien Attack

Uses one of Newton's equations of motion to explore properties of quadratic equations both numerically and graphically.

Years 9 and 10, Levels 1 and 2; Algebra; Coordinate Geometry.

3.7 What's My Line

This worksheet investigates the connection between a table of values, the line on a number plane and the equation of the line.

Years 9 and 10, Levels 1 and 2; Algebra; Coordinate Geometry — Straight Lines.

3.8 Graphing Straight Lines

Using the Y= equation editor to explore the $y = mx + b$ form of a straight line.

Years 9 and 10, Levels 1 and 2; Algebra; Coordinate Geometry.

3.9 Sketching Quadratics — Intercept Method

This activity uses TRACE and ZOOM to determine the intercepts and vertex of a parabola. Students should be able to factorise monic quadratics.

Years 9 and 10, Levels 1 and 2; Algebra; Coordinate Geometry.

3.10 Starburst

A study of straight lines: slope and intercept.

Year 9, Levels 1 and 2; Algebra; Coordinate Geometry.

3.11 Speeding — A Study in Linear Functions

Students learn and apply basic knowledge of linear functions to problems involving speeding tickets.

Year 9, Levels 1 and 2; Algebra; Coordinate Geometry.

3.12 Which Fuel?

An application of linear functions to choosing whether to use petrol or LPG in your car.

Years 9 and 10, Level 1; Algebra; Coordinate Geometry.

3.13 Temperature Conversions

An applications of linear functions to conversion between degrees Celsius and degrees Fahrenheit.

Year 9, Levels 1 and 2; Algebra; Coordinate Geometry.