The Graphics Screen and Accuracy

TI-83/83+

Contents

ADFA **CMA** **T^3**

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At *www.unsw.adfa.edu.au/pems/news/high_school/hsc_activities.html*

- A variety of graphics-calculator activities for Years 9 and 10 — written as part of the CQTP Program.
- TI-83 programs and program information.
- *Using the TI-83/83+* — an introduction to the basic operations, suitable for Years 8–12.
- *Sequences and Series on a TI-83/83+* — basic commands and a variety of problems, suitable for Years 11 and 12.
- *Coordinate Geometry on a TI-83/83+* — basic commands and a variety of problems, suitable for Years 9 and 10.
- *Population Modelling* — a variety of problems from simple exponential growth to Leslie matrices and difference equations, covering Years 7–12.
- *Calculus on a TI-83/83+* — basic commands and a variety of problems, suitable for Years 11 and 12.
- *Matrices on a TI-83/83+* — suitable for Years 11 and 12.
- *Complex Numbers on a TI-83/83+* — suitable for Years 11 and 12.
- *Introduction to Complex Numbers* — complex numbers from the beginning, covering the basic operations, but set in the context of complex numbers as a mathematical structure.
- *Programming a TI-83/83+* — suitable for teachers and keen students.

1 The Graphics Screen

Modified from D. Pence, *Calculus Activities for TI Graphic Calculators*, 2nd ed, PWS Publishing, 1994.

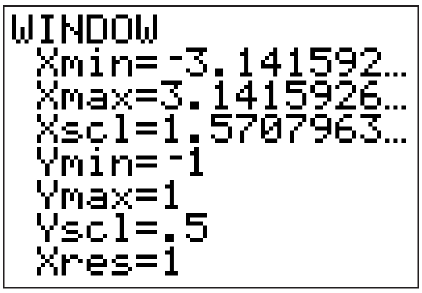
Before graphing a function f on a graphics calculator, you must first specify the range of X values to be considered (i.e. restrict the domain to a finitely bounded interval of numbers) and the range of Y values to be allowed. Setting these ranges defines the scales and locates the coordinate axes on the screen. Generally we will use the estimated range of f as the Y range. However, the true range of f may be an unbounded interval, and so it may not be possible to use it for the Y range.

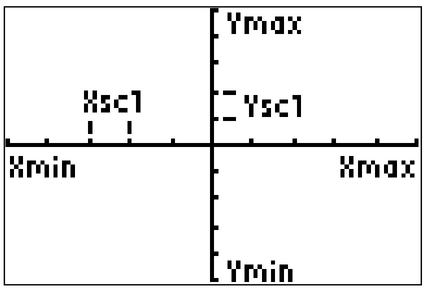
The calculator screen consists of rows and columns of little rectangles, called *pixels*. Compared to a television screen or computer monitor, the pixels on the calculator screen are relatively large. On the TI-83, the screen consists of a grid 96 pixels across and 64 pixels high.[1] You can see the individual pixels by turning up the contrast. To do this, press and release [2nd] and then hold down the up-arrow key. *You will notice numbers in the top right-hand corner which indicate the level of contrast. The greatest contrast is 9. If you need a contrast level of 8 or 9 for normal use, you should think about some new batteries.* Turn the contrast back down again by pressing and releasing [2nd] and then holding down the down-arrow key until you reach the correct setting for you.

Any point (X, Y) on the screen lies in the range

$$\mathrm{Xmin} \leq \mathrm{X} \leq \mathrm{Xmax}, \qquad \mathrm{Ymin} \leq \mathrm{Y} \leq \mathrm{Ymax},$$

where Xmin, Xmax, Ymin, Ymax are set using WINDOW.





The first *column* of pixels on the left has the X coordinate *Xmin*, and the last but one *column* of pixels on the right the X coordinate *Xmax*. Similarly, the *rows* of pixels have equally spaced Y values between *Ymin* and *Ymax*. There is a similar grid of pixels for any computer-drawn coordinate graph, but on a high-resolution monitor or printer the ‘dots’ or pixels are so small that the eye cannot pick them out.

[1] The screen used for graphics is actually 95×63 pixels.

Each pixel has coordinates (the coordinates of the centre of the pixel) that appear at the bottom of the screen as you move the cursor around with the arrow keys. However, each pixel actually represents a region in the plane (i.e. infinitely many points). If a point to be plotted lies somewhere in the region represented by a pixel, the pixel is turned on (darkened). In TRACE mode, the X value at the bottom of the screen is the X coordinate of the pixel, but the Y value is the value of the function at that X value. This is usually different to the Y coordinate of the pixel, but must lie in the range of Y values covered by that pixel.

The X axis will appear on the screen if Ymin $\leq 0 \leq$ Ymax. Scale marks are drawn along the axis at a regular spacing *Xscl*, also set in WINDOW. Similarly, the Y axis will appear if Xmin $\leq 0 \leq$ Xmax, with scale marks spaced by *Yscl*. No scale marks appear if Xscl/Yscl is set to 0. If Xscl/Yscl is too small, the scale marks will run together and the axis will look as though there is a parallel line running right next to it. This is often confused with the graph of the function you are trying to plot.

There are several commands in the ZOOM menu which provide convenient ways to change the WINDOW settings. These commands also contain an implied GRAPH command.

[ZOOM] [6] (ZStandard) gives WINDOW settings $[^-10, 10, 1] \times [^-10, 10, 1]$.

[ZOOM] [7] (ZTrig) gives approximate WINDOW settings of $[^-2\pi, 2\pi, \frac{\pi}{2}] \times [^-4, 4, 1]$, suitable for most trigonometric functions.

[ZOOM] [5] (ZSquare) makes the scales on the X and Y axes the same. For equal scales, (Xmax $-$ Xmin) is $95/63 \approx 1.5$ times (Ymax $-$ Ymin). ZSquare is useful if the shape of the function you are plotting is important, such as a semi-circle.

It is often useful to have a setting in which the coordinates of adjacent columns and rows differ by 0.1, i.e. the pixel widths and heights are both 0.1 . This is achieved by using [ZOOM] [4] (ZDecimal).

2 Function Graphers — Getting the Picture

Modified from T.P. Dick and C.M. Patton, *Student guide to using technology in calculus*, PWS-Kent, Boston, 1992.

A graph provides a powerful interpretational tool by giving us a visual picture of the input–output pairs a function process produces, but it requires much time and effort to prepare graphs by hand. With the availability of computer and calculator graphics technology, we have a much greater opportunity to exploit graphical representations of functions.

Be forewarned: the graphical evidence provided by a machine can be open to perceptual illusions and therefore to misinterpretations. To make intelligent use of graphical tools, it is important to understand their limitations. In other words, getting the most out of graphics technology requires not only knowing how it can be used, but also how it *can't* be used. Let's look at some of the issues you must be concerned with when using graphing technology.

First of all we need to understand how a graph is produced and displayed by a machine. The screen of a calculator or computer is divided into a rectangular grid of small square picture elements called *pixels*. Each pixel has coordinates corresponding to a single point in the plane, but a pixel does not really represent a point. Rather, a pixel represents a small rectangle containing infinitely many points. The specific point given by the coordinates of the pixel may represent the centre or a corner of the pixel, depending on the particular machine or software (*on the TI-83, it's the centre*). If we want the machine to indicate a certain point in the plane, we have to light up or darken the particular pixel containing that point.

The viewing window

Every graphics package on a computer or calculator has a necessarily limited screen. You might think of this screen as a window from which you can view part of the Cartesian plane. By moving this window around the plane, we can focus our attention on various parts of the graph of a function. This window is also a *lens* through which we can obtain both close-up and distant views of the graph by changing scale. Finding the best window locations and scales are navigational skills for finding our way about a function's graph.

Graphical behaviour can be hidden

- by lying beyond the bounds of the viewing window,
- by scale — zooming in obscures global information about the graph; zooming out obscures local information or detail about the graph,
- by numerical limitations — the choice of which pixels to light up or darken is determined by numerical computations, which in turn are subject to the usual round-off, cancellation, underflow and overflow errors that may occur.

Exercise: Graph the function defined by the formula

$$f(x) = \frac{(x^3 - 1)}{(x - 1)},$$

using Y1 and WINDOW parameters $[-6, 5, 2] \times [-2, 10, 2]$. Be careful with brackets.

(a) What is $f(1)$? Check your calculator's reaction to this calculation by evaluating Y1(1) on the TI-83 as follows: [VARS] Y-VARS [1] (Function) [1] (Y1) [(] [1] [)] [ENTER].

(b) Does this problem show up on your graph? Use [TRACE] to investigate. Explain.

(c) Now change the X WINDOW to $[-4.7, 4.7, 2]$ and repeat (b). Why the difference?

(d) Move the cursor using [TRACE] close to, but not at, X = 1 and ZOOM in using [ZOOM] [2].

Do this repeatedly ([TRACE] close to X = 1 then [ZOOM] [2], about 10 times) until an irregularity appears.

ZOOM in several more times. Why do you think this might happen? Look at WINDOW.

The graph of f has a *hole* at $x = 1$, since the function is not defined there. However, a machine plot of this function's graph could have several different appearances near $x = 1$:

- the graph may appear to be continuous if $x = 1$ falls in between the X coordinates of two adjacent pixels (but not close enough to either to have the function value affected noticeably by numerical imprecision);
- there may be a missing pixel if the X coordinate of a pixel is exactly 1;
- there may be a jagged jump or spike if the X coordinate of a pixel is very close, but not equal, to 1, due to numerical imprecision.

You are more likely to observe the visual effects of numerical imprecision at small scalings. Even a continuous function's machine-plotted graph may break apart under repeated zooms because the function cannot be calculated accurately enough.

Exercise: Graph the function $(2\cos(x) - 2 + x^2)/x^4$ using a WINDOW of

(a) $[-10, 10, 0] \times [-0.1, 0.2, 0]$

(b) $[-0.001, 0.001, 0] \times [-0.1, 0.2, 0]$.

What do you observe?

This is another example of numerical instability — the calculator cannot calculate $\cos(x)$ accurately enough when its argument is very close to 0.

The dimensions of the viewing window are specified by the WINDOW parameters *Xmin*, *Xmax*, *Ymin*, *Ymax*. We also know that the TI-83 graphing screen is 95 pixels wide by 63 pixels high. (The actual screen is 96×64, but only 95×63 is used for graphing.) From these numbers we can calculate the dimensions of a pixel.

Example: Suppose we specify WINDOW parameters $[-10, 10] \times [-5, 5]$. Find the dimensions of a pixel.

On the TI-83, the coordinates of a pixel correspond to the centre of the pixel. The pixels are (effectively) touching, so that each interval between adjacent pixels is equal to the width of a pixel. Since there are 95 pixels across the width of the graphing screen, there will be 94 pixel-width intervals between the centre of the leftmost pixel representing *Xmin* and the centre of the rightmost pixel (of the 95) representing *Xmax*. Therefore the *width* of a pixel is

$$\Delta\mathrm{X} = \frac{\mathrm{Xmax}–\mathrm{Xmin}}{94} = \frac{10 - (-10)}{94} = \frac{20}{94} \approx 0.2128.$$

Similarly, since there are 63 pixels vertically, the *height* of each pixel is

$$\Delta\mathrm{Y} = \frac{\mathrm{Ymax}–\mathrm{Ymin}}{62} = \frac{5 - (-5)}{62} = \frac{10}{62} \approx 0.1613.$$

ΔX and ΔY can be accessed through the VARS Window menu. These values are calculated whenever you plot a graph.

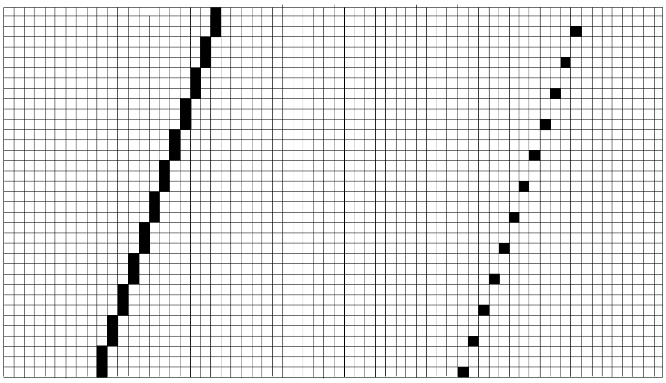
Exercise: If the viewing window is $[-4.7, 4.7] \times [-3.1, 3.1]$, find the dimensions of each pixel.

This is often a useful WINDOW to use; it is set automatically by ZDecimal in ZOOM.

When graphing a function, the calculator starts with the first (leftmost) column of pixels. It computes the ordered pair (X, f(X)) using the X coordinate of that column (Xmin) as the value of X, and darkens the pixel in that column whose Y coordinate is closest to f(X), provided f(X) is within the vertical range of the window, i.e. between Ymin and Ymax. This process is then repeated for each column of pixels from left to right.

The calculator may be in either *Dot* or *Connected* mode (MODE key on the TI-83: use the cursor and [ENTER] to select).[2] In *Dot mode*, the calculator will darken at most one pixel in each column (so the function graph on screen will pass the vertical line test). In *Connected* mode, the function grapher will darken additional pixels to give the visual perception of an unbroken graph. The figure below shows two calculator graphs of the same line; one is plotted in *Connected* mode and the other is plotted in *Dot* mode.

In either case, note that the calculator graph of a function is simply a finite collection of darkened pixels, whereas the true graph generally consists of infinitely many points.



Exercise: Darken the screen by pressing and releasing [2nd] and then holding down the up arrow until you can see individual pixels. Plot Y = 5X using [ZOOM] [6] (standard axes). Compare the graphs using *Dot* and *Connected* modes.

[2]On the TI-83, you can select the graph type for individual functions by moving the cursor to the left of the Yn in the [Y=] menu and scrolling through the line types using [ENTER]. The first (default) line type is the connected line, the last line type the dotted line. In *Dot* mode, the dotted line becomes the default, but you can still change the line type for individual functions.

3 Significant Digits and Calculations

When we write down a numeric answer to a problem, we have to make some decision as to how many digits we put in the answer. Firstly we have to decide how accurate our answer is — this will depend on the accuracy of the data we use in our calculations and on whether we introduce any further loss of accuracy by our method of calculation. These considerations are discussed below. Secondly, we have to decide what is a *sensible* number of digits to put in our answer. We wouldn't give the distance to a star to a number of digits that takes us down to millimeters, even if we knew it that accurately.

In specifying the accuracy of an answer, we usually give the number of decimal places (DP) or the number of significant digits (SD) that we think are appropriate. We will use significant digits in this course, because it fits in well with scientific notation for specifying numbers, for example 3.56×10^{-5}.

Examples

1. 0.0036 has 2 SD — leading zeros are not significant. We could also write this number as 3.6×10^{-3}, making the number of significant digits clear.

2. 2 may have one SD or may also be an exact number and so implicitly have an infinite number of SD.

3. 2. has 1 SD.

4. 2.00 has 3 SD — trailing zeros after the decimal point are significant.

5. 240 000 000 has 2 SD — trailing zeros before a decimal point are not significant unless specified as being so. Write as 2.4×10^8.

6. Be careful with rounding. If your answer is an approximation, you should specify the number of significant digits which are accurate.

$$\begin{aligned} \pi &\approx 3.141592654 \quad \text{(10 SD)} \\ &\approx 3.14 \quad \text{(3 SD: rounding down)} \\ &\approx 3.142 \quad \text{(4 SD: rounding up)} \\ &\approx 3.1416 \quad \text{(5 SD: rounding down)} \end{aligned}$$

There are at least three sources of concern about significance of digits in an answer.

- **The accuracy of available data**

  You should not expect more significant digits in any answer than there are in the least accurate input to the calculation. *Calculation steps never add significant digits, though your calculator will happily add digits!*

  Example: A population of 240 000 000 grows by 2% per year for 3 years. What's the population after 3 years?

  Numerically, the answer is $240\,000\,000 \times (1.02)^3 = 254\,689\,920$. However, the original number had only 2 SD (2 and 4) and possibly 3 — we don't really know about the first 0. The best answer to the question is "about 250 000 000", or possibly "about 255 000 000" if we thought the first 0 was significant. The answer 254 689 920 is definitely wrong.

- **The finite precision of your calculating device**

  Subtraction of nearly equal numbers can be a real significance killer

  Example: If we subtract two numbers that only differ in the tenth digit, the answer has only one significant digit.

  Adding or subtracting numbers that are very different in magnitude can also lead to inaccuracy

  Exercise: We all know that $A + B - A = B$.
  Store 10^6 in memory A.[3]
  Store $\sqrt(2) \times 10^{-5}$ (correct $-$ sign?) in B.
  Evaluate $A + B - A$.
  Repeat[4] with $A = 10^7$, 10^8, 10^9. Explain.

- **Loss of significance due to the way we manipulate numbers**

  Don't discard digits in an intermediate result. The only time you should round off is at the end of a calculation. Preferably use your calculator to do the calculation all in one go — it keeps 14 digits.

[3] [EE] [6] [STO] [ALPHA] [A] [ENTER]

[4] Remember the [ENTRY] key.