Complex Numbers on a CFX-9850GB

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At *www.unsw.adfa.edu.au/pems/news/high_school/hsc_activities.html*

- A variety of graphics-calculator activities for Years 9 and 10 — written as part of the CQTP Program for Sharp, Casio and TI calculators.
- *Using the CFX-9850GB* — an introduction to the basic operations, suitable for Years 8–12.
- *The Graphics Screen and Accuracy* — information to help you understand the graphical and numerical limitations of a graphics calculator.
- *Coordinate Geometry on a CFX-9850GB* — basic commands and a variety of problems, suitable for Years 9 and 10.
- *Population Modelling* — a variety of problems from simple exponential growth to Leslie matrices and difference equations, covering Years 7–12.
- *Sequences and Series on an CFX-9850GB* — basic commands and a variety of problems, suitable for Years 10–12.
- *Matrices on an CFX-9850GB* — suitable for Years 11 and 12.
- *Calculus on an CFX-9850GB* — suitable for Years 11 and 12.
- *Introduction to Complex Numbers* — complex numbers from the beginning, covering the basic operations, but set in the context of complex numbers as a mathematical structure.

The program CMPXROOT listed in these notes can be found at the above web site. You will need a PC-Link Kit FA-123 to copy the program from your computer to your calculator.

1 Setting Complex Mode

The CFX-9850GB is always in Complex mode — there is no need to select anything. The calculations are all done on the RUN screen ([MENU] [1]).

To access the complex operations, press [OPTN] [F3] (CPLX).

We shall use $z_1 = 1 + 2i$ and $z_2 = 3 - i$ in our examples. i is [F1] in the CPLX menu.

Complex numbers can be stored in the same way as ordinary numbers. Store z_1 in memory A: $1 + 2i$ [→] A; z_2 in memory B: $3 - i$ [→] B.

2 Basic Operations

2.1 Addition and subtraction

Just as you would expect.

$1 + 2i + 3 - i = 4 + i$ or $A + B = 4 + i$

$1 + 2i - (3 - i) = -2 + 3i$ or $A - B = -2 + 3i$

2.2 Multiplication and division

Again as you would expect. Implied multiplication works too.

$(1 + 2i)(3 - i) = 5 + 5i$ or $AB = 5 + 5i$

$(1 + 2i) \div (3 - i) = 0.1 + 0.7i$ or $A \div B = 0.1 + 0.7i$

2.3 Conjugation

Finding the complex conjugate.

$\bar{z}_1 = \text{Conjg}\,(1+2i) = 1-2i$ *Conjg* is [F3]; brackets are necessary unless the number is pure real or pure imaginary.

2.4 Real part

$\text{Re}(z_1) = \text{ReP}\,(1 + 2i) = 1$ *ReP* is [F5]; again brackets are necessary.

2.5 Imaginary part

$\text{Im}(z_1) = \text{ImP}\,(1 + 2i) = 2$ *ImP* is [F6]; again brackets are necessary.

2.6 Modulus

Sometimes called length or absolute value.

$|z_1| = \text{AbS}\,(1 + 2i) = \sqrt{1^2 + 2^2} = \sqrt{5} \approx 2.236$ *AbS* is [F2]; include brackets.

3 Polar Form

The polar form (r, θ) of a complex number can not be entered directly into the CFX-9850GB, but must first be converted to Cartesian form $x + iy$. See Section 3.2. The calculator will tell you the values for x and y, which you then have to enter manually as $x + iy$.

3.1 Modulus and angle

The CPLX operation *AbS*, used in Section 2.6, finds the modulus r, while *Arg* ($\boxed{\text{F3}}$) finds the angle θ, of a complex number given in Cartesian form. The angle is given in radians or degrees, depending on the angle setting in $\boxed{\text{SETUP}}$.

$\text{Arg}(z_1) = \text{Arg}\,(1 + 2i) \approx 63.435° \approx 1.107$ radians.

3.2 Conversion between forms

To convert complex numbers from Cartesian or rectangular form to polar form and vice versa, use the conversions in the $\boxed{\text{ANGL}}$ menu — $\boxed{\text{OPTN}}$ $\boxed{\text{F6}}$ $\boxed{\text{F5}}$. Press $\boxed{\text{F6}}$ again to see *Pol(* and *Rec(*.

In *Radian* mode, converting $1 + i$ to polar form:

$\text{Pol}\,(1, 1)$ $\boxed{\text{EXE}}$ gives $r \approx 1.414213562$ ($\sqrt{2}$) and $\theta \approx 0.7853981634$ ($\pi/4$).

Converting $(r, \theta) = \left(\sqrt{2}, \pi/4\right)$ to Cartesian or rectangular form:

$\text{Rec}(\sqrt{2}, \pi \div 4)$ $\boxed{\text{EXE}}$ gives $x = 1$ and $y = 1$, so the Cartesian form is $1 + i$.

4 Powers and Roots

4.1 Powers

Only the $\boxed{x^2}$ and $\boxed{x^{-1}}$ keys work. You can't use the $\boxed{\wedge}$ key at all.

$(1 + 2i)^2 = -3 + 4i$.

$(1 + 2i)^{-1} = 0.2 - 0.4i$.

4.2 Roots

Only the square-root key $\boxed{\sqrt{\ }}$ works, and it only gives you one root of a complex number. To find all the roots,[1] you can use the program CMPXROOT.

[1] A complex number has n nth roots (n a positive integer).

4.3 The CMPXROOT program

Download from *www.ma.adfa.edu.au/Events/WorkExperience/index.html*

Calculates the Cartesian form $x + iy$ of each of the N Nth roots of the complex number $A + iB$.

Use: Run the program. Input values for A, B and N (positive integer) when prompted. The program displays the roots in Mat A, with the real parts x in the first column and the imaginary parts y in the second column.

Example: The cube roots (N = 3) of $1 + 2i$ are $1.220 + 0.472i$, $-1.018 + 0.820i$ and $-0.201 - 1.292i$, all rounded to three decimal places.

5 Exercises

$$z_1 = 3 + 4i \qquad z_2 = 2 + 3i \qquad z_3 = \sqrt{2}\,\mathrm{cis}\,(\pi/4) \qquad z_4 = \mathrm{cis}\,(\pi/2)$$

Find

1. $z_1 + z_2$
2. $2z_1 + 3z_2$
3. $z_1 - z_2$
4. $4z_1 - 2z_2$
5. $z_1 z_2$
6. z_1/z_2
7. $\bar{z}_1$
8. $z_1\bar{z}_1$
9. $|z_1|^2$
10. $\mathrm{Re}(z_1)$
11. $\mathrm{Im}(z_2)$
12. z_1^2
13. z_1^4
14. $\sqrt{z_1}$
15. $z_3 z_4$ in polar form
16. z_3/z_4 in polar form
17. $\sqrt{z_4}$ in polar and Cartesian form
18. z_1 in polar form
19. z_3 in Cartesian form
20. z_4 in Cartesian form

6 Answers to Exercises

1. $5 + 7i$
2. $12 + 17i$
3. $1 + i$
4. $8 + 10i$
5. $-6 + 17i$
6. $18/13 - i/13 \approx 1.3846 - 0.0769i$
7. $3 - 4i$
8. 25
9. 25
10. 3
11. 3
12. $-7 + 24i$
13. $-527 - 336i$
14. $2 + i$ and $-2 - i$
15. $(r, \theta) = \left(\sqrt{2}, 3\pi/4\right) \approx (1.4142, 2.3562)$
16. $(r, \theta) = \left(\sqrt{2}, -\pi/4\right) \approx (1.4142, -0.7854)$
17. $(r, \theta) = (1, \pi/4)$ and $(1, 5\pi/4)$ or $\pm(1 + i)/\sqrt{2}$
18. (5, 0.927295218)
19. $1 + i$
20. i