Complex Numbers on an EL-9650/9900

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Contents

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At *www.unsw.adfa.edu.au/pems/news/high_school/hsc_activities.html*

- A variety of graphics-calculator activities for Years 9 and 10 — written as part of the CQTP Program for Sharp, Casio and TI calculators.
- *Using the EL-9650/9900* — an introduction to the basic operations, suitable for Years 8–12.
- *The Graphics Screen and Accuracy* — information to help you understand the graphical and numerical limitations of a graphics calculator.
- *Coordinate Geometry on an EL-9650/9900* — basic commands and a variety of problems, suitable for Years 9 and 10.
- *Population Modelling* — a variety of problems from simple exponential growth to Leslie matrices and difference equations, covering Years 7–12.
- *Sequences and Series on an EL-9650/9900* — basic commands and a variety of problems, suitable for Years 10–12.
- *Matrices on an EL-9650/9900* — suitable for Years 11 and 12.
- *Calculus on an EL-9650/9900* — suitable for Years 11 and 12.
- *Programming an EL-9650/9900* — suitable for teachers and keen students.
- *Introduction to Complex Numbers* — complex numbers from the beginning, covering the basic operations, but set in the context of complex numbers as a mathematical structure.

The program CMPXROOT listed in these notes can be found at the above web site. You will need a PC-Link Kit CE-LK2 to copy the program from your computer to your calculator.

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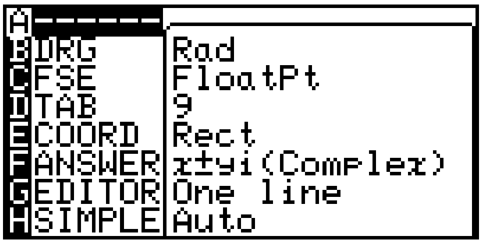
1 Setting Complex Mode

Press SETUP F (ANSWER) and select the Cartesian form $x \pm yi$ *(Complex)* with the cursor and ENTER. Set the other options as shown in the figure. Press the Home key (the one with $+ - \times \div$ on it) to return to the home screen.



We shall use $z_1 = 1 + 2i$ and $z_2 = 3 - i$ in our examples.

i is 2ndF $\cdot$.

Complex numbers can be stored in the same way as ordinary numbers. Store z_1 in memory A: $1 + 2i$ STO A; z_2 in memory B: $3 - i$ STO B.

2 Basic Operations

2.1 Addition and subtraction

Just as you would expect.

$1 + 2i + 3 - i = 4 + i$ or $A + B = 4 + i$

$1 + 2i - (3 - i) = -2 + 3i$ or $A - B = -2 + 3i$

2.2 Multiplication and division

Again as you would expect. Implied multiplication works too.

$(1 + 2i)(3 - i) = 5 + 5i$ or $AB = 5 + 5i$

$(1 + 2i) \div (3 - i) = 0.1 + 0.7i$ or $A \div B = 0.1 + 0.7i$

2.3 Conjugation

Finding the complex conjugate.

$\bar{z}_1 = \text{conj}(1 + 2i) = 1 - 2i$ *conj* is MATH H (COMPLX) 1

2.4 Real part

$\text{Re}(z_1) = \text{real}(1 + 2i) = 1$ *real* is MATH H 2

2.5 Imaginary part

$\text{Im}(z_1) = \text{image}(1 + 2i) = 2$ *image* is [MATH] [H] [3]

2.6 Modulus

Sometimes called length or absolute value.

$|z_1| = \text{abs}(1 + 2i) = \sqrt{1^2 + 2^2} = \sqrt{5} \approx 2.236$ *abs* is [MATH] [H] [4]

3 Polar Form

The polar form (r, θ) of a complex number is entered into the EL-9650/9900 as $r\angle\theta$, where r is the modulus or length and θ is the angle. The separator $\angle$ is [2ndF] [0].

For manual calculations, we write complex numbers in polar form as $r\big(\cos(\theta) + i\sin(\theta)\big)$ $\big($abbreviated to $r\text{cis}(\theta)\big)$ or as $re^{i\theta}$.

3.1 Modulus and angle

The MATH COMPLX operations *abs* and *arg* can be used to to find the modulus and angle respectively of a complex number in either Cartesian or polar form.

Similarly, *real* and *image* can be used to find the real and imaginary parts respectively of a number in either form, and *conj* its complex conjugate.

3.2 Form of output

Complex numbers can be input in either Cartesian or polar form, no matter what the SETUP ANSWER selection, but the output form will depend on whether $x \pm yi$ (Cartesian form) or $r\angle\theta$ (polar form) is selected.

3.2.1 Cartesian ($x \pm yi$) form

Make sure the calculator is set to *Radian* mode (in SETUP).

Input $\big(\sqrt{2}\big) \angle \big(\pi \div 4\big)$ and press [ENTER] to give $1 + i$.

The brackets here are necessary, because the $\angle$ operation precedes both the square root and divide operations — see *Precedence of Calculations* in the calculator guidebook.

To input an angle in degrees while in Radian mode, use the degree symbol [MATH] [E] (ANGLE) [1]: $\big(\sqrt{2}\big) \angle \big(45°\big) \quad \rightarrow \quad 1 + i$.

The second set of brackets are again necessary, because the $\angle$ operation precedes the degree operation.

3.2.2 Polar ($r\angle\theta$) form

Set polar form $r\angle\theta$ using SETUP ANSWER. Make sure you are still in *Radian* mode.

Enter the number $1+i$ (Cartesian form) and press ENTER.

$1+i \quad \to \quad 1.414213562\angle 0.785398163.$

If you repeat this in *Degree* mode,

$1+i \quad \to \quad 1.414213562\angle 45,$

i.e. the angle is displayed in degrees.

3.3 Form of input

In carrying out calculations using complex numbers, you can input the numbers in any form, Cartesian or polar or mixed. The SETUP ANSWER setting only determines the output form of the answer. In *Radian* mode,

$$\begin{aligned} 1+i+\left(\sqrt{2}\right)\angle\left(45^\circ\right) &= 2.828427125\angle 0.785398163 \quad \text{Polar form} \\ &= 2+2i \quad \text{Cartesian form.} \end{aligned}$$

3.4 Conversion between forms

To convert complex numbers from one form to the other, use the conversions in the MATH CONV menu.

In *Radian* mode, converting $1+i$ to polar form:

$xy \to r(1,1)$ ENTER gives $r = 1.414213562$ $(\sqrt{2})$,

$xy \to \theta(1,1)$ ENTER gives $\theta = 0.785398163$ $(\pi/4)$.

Converting $\left(\sqrt{2}, \pi/4\right)$ to Cartesian form:

$r\theta \to x(\sqrt{2}, \pi \div 4)$ ENTER gives $x = 1$,

$r\theta \to y(\sqrt{2}, \pi \div 4)$ ENTER gives $y = 1$.

4 Powers and Roots

4.1 Powers

Integer powers (and x^{-1}) work as you would expect. Watch brackets when using the polar form.

$(1+2i)^4 = -7-24i$.

$\left((\sqrt{2})\angle(\pi \div 4)\right)^4 = 4\angle\pi = -4.$

4.2 Roots

Unfortunately you only get one root of a complex number when you use square root, cube root, xth root and fractional powers. To find all the roots,[1] you can use the program CMPXROOT, which calculates and plots the nth roots of a given complex number.

4.3 The CMPXROOT program

Download from *www.ma.adfa.edu.au/Events/WorkExperience/index.html*

Displays the rectangular values (x, y) and plots the Nth roots of the complex number $A+iB$, i.e. the points $Z = (A+iB)^{\frac{1}{N}}$, on the Argand diagram. The roots all lie on a circle which is drawn. Lines are drawn from the origin to each root to show where the root is and to make the symmetry of the roots more obvious. Note that if a root Z lies on a co-ordinate axis, you won't see it (unless you turn the axes off), but it should be obvious from symmetry.

Use: Run the program. Input values for A, B and N (positive integer) when prompted. The program displays each root and then plots it. Press [ENTER] to continue at each step and [ENTER] to finish after the final root is plotted. Press [GRAPH] and move the cursor around the plot using the arrow keys to see rectangular (x, y) values if [FORMAT] CURSOR is set to *RectCoord* or polar (R, θ) values if [FORMAT] CURSOR is set to *PolarCoord*.

The roots are stored in list L1 if you need them later.

Example: The cube roots (N = 3) of $1+2i$ are $1.220+0.472i$, $-1.018+0.820i$ and $-0.201-1.292i$, all rounded to three decimal places.

[1] A complex number has n nth roots (n a positive integer).

5 Exercises

$$z_1 = 3 + 4i \qquad z_2 = 2 + 3i \qquad z_3 = \sqrt{2}\,\text{cis}\,(\pi/4) \qquad z_4 = \text{cis}\,(\pi/2)$$

Find

1. $z_1 + z_2$
2. $2z_1 + 3z_2$
3. $z_1 - z_2$
4. $4z_1 - 2z_2$
5. $z_1 z_2$
6. z_1/z_2
7. $\bar{z}_1$
8. $z_1\bar{z}_1$
9. $|z_1|^2$
10. $\text{Re}(z_1)$
11. $\text{Im}(z_2)$
12. z_1^2
13. z_1^4
14. $\sqrt{z_1}$
15. $z_3 z_4$ in polar form
16. z_3/z_4 in polar form
17. $\sqrt{z_4}$ in polar and Cartesian form
18. z_1 in polar form
19. z_3 in Cartesian form
20. z_4 in Cartesian form

6 Answers to Exercises

1. $5 + 7i$
2. $12 + 17i$
3. $1 + i$
4. $8 + 10i$
5. $-6 + 17i$
6. $18/13 - i/13 \approx 1.3846 - 0.0769i$
7. $3 - 4i$
8. 25
9. 25
10. 3
11. 3
12. $-7 + 24i$
13. $-527 - 336i$
14. $2 + i$ and $-2 - i$
15. $\sqrt{2}\angle(3\pi/4) \approx 1.4142\angle 2.3562$.
16. $\sqrt{2}\angle(-\pi/4) \approx 1.4142\angle - 0.7854$
17. $1\angle(\pi/4)$ and $1\angle(5\pi/4)$ or $\pm(1 + i)/\sqrt{2}$
18. $5\angle 0.927295218$
19. $1 + i$
20. i