Graphics Calculator Resources for Years 9 and 10

Activity	*Alien Attack*
Year Group	9, 10
Level	1, 2
Strand	Algebra
Sub-Strand	Coordinate Geometry
Authors	Vanessa Moore and Sherry Morton, Lesson 11 of *Graphing Calculators in Mathematics Grades 7–12* from the Center of Excellence for Science and Mathematics Education, University of Tennessee (*cesme.utm.edu/resources/math/grcalc/toc.html*). Modified by Peter McIntyre (*p.mcintyre@adfa.edu.au*).
Calculators	Sharp EL-9650/9900
Description	Uses one of Newton's equations of motion to explore properties of quadratic equations both numerically and graphically.

Alien Attack

When an object is propelled straight upwards, gravity slows it down and eventually pulls it back to Earth. The graph of height vs time is a parabolic curve, even though the path of the object is straight up and down.

The height of the object as a function of time is given by one of Newton's equations of motion,

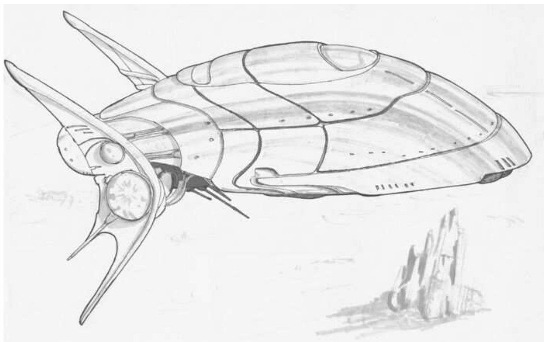
$$h = h_i + v_i t - 0.5gt^2,$$

where h is the height above the ground, t is time, h_i is the initial or starting height, v_i is the initial velocity and g is the acceleration due to gravity (a constant).

If we use SI units of height in metres and time in seconds, $g \approx 9.8\,\text{m/s}^2$.

The scene

An alien spaceship is hovering above the city at a height of 100 m. A giant slime blob is housed in a missile-like container. The alien ship launches the container straight up into the air at a velocity of 500 m/s and vanishes immediately into hyperspace. What happens?



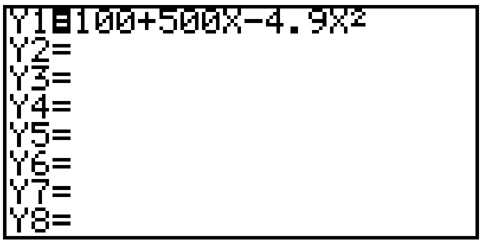
www.sheriftariq.org/markers/elegant_alienship.jpg

To answer this question, you need to put the equation for height into your calculator.

(a) First write the equation for h in the space below, using the given values for h_i, v_i and g. You can probably also work out 0.5×9.8 to shorten the equation.

(b) The calculator uses X for the (independent variable) time and Y for the (dependent variable) height. Write the equation for h in terms of Y and X in the space below.

(c) Now press [Y=] (top left).
Enter your equation into Y1 just as you have written it above. The X key is the one labelled [X/θ/T/n] in the fifth row, sixth column.
Make sure you use the minus sign [−], not the change sign [(−)].

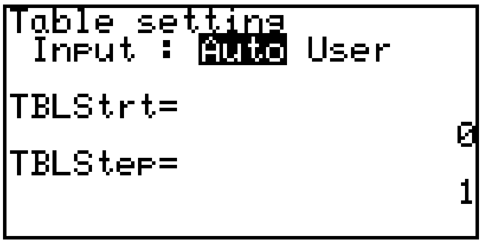


Now you will use TABLE to generate a table of values of the height function you have just entered.

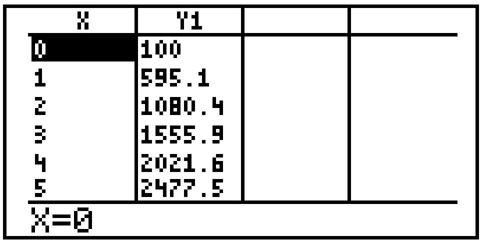
(d) To tell the calculator which X values you want in the table, press [TBLSET] ([2ndF] [TABLE]).

Set *TBLStrt* = 0 and *TBLStep* = 1, so that the table of values will start at time X = 0 and increment in steps of 1 s.

If it is not already selected, select *Auto* with the cursor and [ENTER].



(e) Press [TABLE].



Now answer each of the following questions in the space below the question.

1. What is the height of the container after 25 seconds?

2. Is the container going up or down after 25 seconds? How do you know?

3. What is the maximum height the container reaches?

4. After how many seconds does the container reach its maximum height?

5. How accurate is your value in Question 4? Look at the table and decide between which two X values you are sure the exact answer lies.

The exact time to maximum height is greater than ______, *but less than* ______.

6. To obtain a more accurate answer to Question 4, set *TBLStep* in [TBLSET] to 0.1. You may want to change *TBLStrt* too. Scroll in the Y1 column of the table to see the highlighted Y value with more digits at the bottom of the screen.

Find the time to maximum height, *accurate to one decimal place*, and the corresponding maximum height. *Hint*: You may need to change *TBLStep* more than once.

7. When the container hits the ground, the slime blob will envelop the city. How long does our local superhero[1] ____________________ have to come to the rescue? *Hint*: Use the table to answer the question. Tenths of seconds are vital here.

Next we will look at graphs of height vs time to answer some more questions.

You already have the right function to graph because you used it for the table. You need to set a suitable WINDOW so the graph appears on your screen. Press [WINDOW].

8. What quantity does X represent here? Based on your explorations using TABLE, what are suitable values for *Xmin* and *Xmax*? What does Y represent here? What are suitable values for *Ymin* and *Ymax*?

Enter these into your calculator. Set *Xscl* (the distance between tick marks on the X axis) to 10 and *Yscl* to 5000. Press [TRACE].

Change your WINDOW values if necessary and re-graph so that the graph fills the screen.

9. Sketch your graph below, being sure to label the axes with what they actually represent and giving some idea of scale (one or two values on each axis).

[1] Supply an appropriate name.

10. What is the name of the point on the parabola that corresponds to maximum height? What are the approximate coordinates of this point? (Use the left- and right-arrow keys to determine this.) Does this agree with what you found using the table?

On your graph, you will notice that the cursor coordinates cover the X axis. To fix this, change *Ymin* in [WINDOW] to about −1500 (which minus key?).
Press [TRACE].

11. Find the approximate time that the container hits the ground using the cursor in TRACE mode. How accurate is your answer? Does it agree with the answer you found using a table?

The exact time to ground is greater than __________, *but less than* __________.

12. A point at which a graph crosses the x axis (has value 0) is called an *x intercept* or *zero*.

The time taken for the container to return to the ground is an x intercept of the height graph. In Question 11, you found an approximate value for this.

You will now use the *X_Incpt* operation on the EL-9650/9900 to find the time to ground more accurately.

Press [CALC] ([2nd] [TRACE]) and select *X_Incpt*. The calculator will find an x intercept. Keep selecting *X_Incpt* until you find the x intercept you want.

When does the container hit the ground? Give your answer to a *sensible* number of decimal places.

13. Does our superhero save the city? Finish the story.

Teacher's Notes

The equation for height as a function of time is

$$h(t) = 100 + 500t - 4.9t^2,$$

with the corresponding calculator equation and table of values shown in the figures.

1. What is the height of the container after 25 seconds?

Scroll down the table to find that $h(25) = 9537.5$, *so the container is at a height of 9537.5 m after 25 s.*

2. Is the container going up or down after 25 seconds? How do you know?

The container is still going up after 25 s, because the height is increasing with time then.

3. What is the maximum height the container reaches?

Scroll down the table to find that the maximum height the container reaches is apparently 12855 m.

4. After how many seconds does the container reach its maximum height?

According to the table, the container reaches maximum height after 51 s.

5. How accurate is your value in Question 4? Look at the table and decide between which two X values you are sure the exact answer lies.

The exact time to maximum height is greater than 50 s, but less than 52 s.

The table values are for integer numbers of seconds. We can only say for sure that the maximum time lies between 50 s and 52 s, with a best estimate of 51 s. You could draw a head-up parabola and discuss where the three points in the table near the maximum might lie, in particular that the highest point may not lie right at the vertex.

6. Using TABLE to obtain more accurate answers.

By changing TBLStep *in* TBLSET *to 0.1, we can see the height every 0.1 s. It's a good idea to make* TBLStrt = 50 *so you don't have to scroll through too many values.*

X	Y1
50	12850
50.1	12851
50.2	12851.8
50.3	12852.6
50.4	12853.2
50.5	12853.8

Y1=12850.951

Find the time to maximum height, *accurate to one decimal place*, and the corresponding maximum height.

After changing TBLStep *in* *TBLSET* *to 0.1, we find the time to maximum height lies between 50.9 s and 51.1 s, so we can only say the answer is 51 s, accurate to zero decimal places. Changing* TBLStep *in* *TBLSET* *to 0.01 (and* TBLStrt *to 51.0 say), we find the time to maximum height lies between 51.01 s and 51.03 s, both 51.0 s, accurate to one decimal place.*

The corresponding maximum height is 12855 m, rounded to the nearest metre.

7. When the container hits the ground, the slime blob will envelop the city. How long does our local superhero ____________________ have to come to the rescue? *Hint*: Use the table to answer the question. Tenths of seconds are vital here.

Use the same method to find when $h = 0$ *as you did to find the maximum value of* h*: with* TBLStrt $= 1$, *scroll down the table until Y1 changes sign; use* TBLStep $= 0.1$ *to find the time accurate to one decimal place.*

The time at which the container hits the ground lies between 102.2 s and 102.3 s, with a best estimate of 102.2 s, accurate to the nearest tenth of a second.

Next we will look at graphs of height vs time to answer some more questions.

Here it would be a good idea to run the SLIME program on the overhead projector and discuss what you see. See page 10 for details.

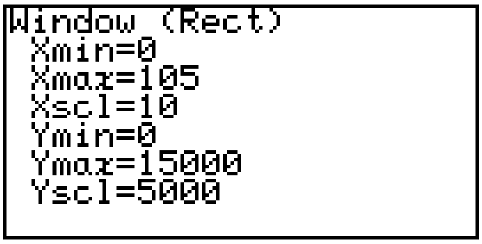
You already have the right function to graph because you used it for the table. You need to set a suitable WINDOW so the graph appears on your screen. Press WINDOW.

8. What quantity does X represent here? Based on your explorations using TABLE, what are suitable values for *Xmin* and *Xmax*? What does Y represent here? What are suitable values for *Ymin* and *Ymax*?

X represents time, so that Xmin $= 0$ *is the starting time. From our results using the table, we know that the container hits the ground after about 102 s, so set* Xmax $= 105$.

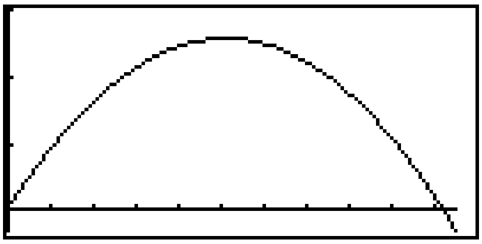
Y represents height above the ground, so that Ymin $= 0$ *is ground level. Again from our table results, we know maximum height is about 12855, so set* Ymax $=$ 15000 *say.*

Enter these into your calculator. Set *Xscl* (the distance between tick marks on the X axis) to 10 and *Yscl* to 5000.



Press GRAPH.

9. Sketch your graph below, being sure to label the axes with what they actually represent and giving some idea of scale (one or two values on each axis).[2] TRACE might be useful here too.



10. What is the name of the point on the parabola that corresponds to maximum height? What are the approximate coordinates of this point? (Use the left- and right-arrow keys to determine this.) Does this agree with what you found before?

The point on the parabola that corresponds to maximum height is called the vertex. Its coordinates are approximately $(51, 12855)$,[3] *roughly the values we found from the table for maximum height.*

For better students, insert *Finding the equation of the parabola* (page 11) here.

On your graph, you will notice that the cursor coordinates cover the X axis. To fix this, change *Ymin* in WINDOW to about minus ((−) key) a tenth of *Ymax*, i.e. to −1500. Press TRACE.

11. Find the approximate time that the container hits the ground using the cursor in TRACE mode. How accurate is your answer?

The time to ground is greater than about 101.6 s, but less than about 102.8 s.

These numbers, the X coordinates of adjacent pixels, will vary a little, depending on what values you put in WINDOW. The best estimate is 102 s.

12. A point at which a graph crosses the x axis (has value 0) is called an *x intercept* or *zero*.

The time taken for the container to return to the ground is an x intercept of the height graph. In Question 11, you found an approximate value for this.

We will now use *X_Incpt* to find the time to ground more accurately.

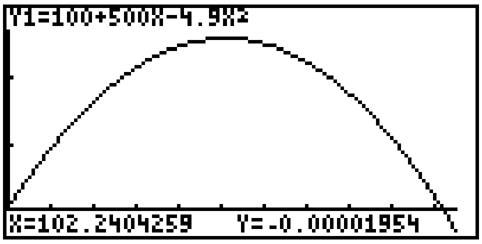
Press CALC (2ndF TRACE) and select *X_Incpt*. Keep selecting *X_Incpt* until you find the x intercept you want (there's only one here if you have set the WINDOW correctly).

[2]Note that *Ymin* has been changed to −1500 here so that the cursor coordinates don't obscure the X axis when you press TRACE.

[3]Select *Value* from the CALC menu, type 51 and press ENTER to go to this point.

(c) When does the container hit the ground? Give your answer to a sensible number of decimal places.

The container hits the ground at 102.24 s, rounded to 2 decimal places.



Further questions

What happens if the initial velocity is changed to

(a) 1500 m/s? *Answer*: max at (153.06 s, 114896 m); reaches the ground at 306.19 s.

(b) 3000 m/s? *Answer*: max at (306.12 s, 459284 m); reaches the ground at 612.28 s.

(c) 5000 m/s? *Answer*: max at (510.20 s, 1275610 m); reaches the ground at 1020.43 s.

The SLIME program

This could be done with the whole class just before starting to plot the graph, that is in the middle of page 7.

The SLIME program shows the action in real time.

- Press [PRGM] and select the SLIME program. The program plots two graphs simultaneously.
- Once the graphs are finished, you can re-live the action in slow motion. Move forward in time with the right arrow, backward in time with the left arrow and between graphs with the up/down arrows. Try this to see corresponding points on the graphs. The cursor coordinates are shown at the bottom of the screen, the relevant ones being time T and height Y.

  Why are the two graphs different? What does each one represent?

  The left-hand graph is a plot of distance versus time. The right-hand graph is the actual trajectory.
- Once you have finished looking at the graphs, press [ENTER] to stop the program. To rerun it, just press [ENTER] again.

The SLIME program is available for download at *www.ma.adfa.edu.au* under *High School and College Activities*. Other resource materials can also be found here.

Finding the equation of the parabola

This follows on after Question 10. Suitable for better students.

If we drew a vertical line through this point, we would have an axis of ____________ of the parabola. What is the equation of this line? What is happening to the slime-blob container on the left-hand side of this line? on the right-hand side?

Write the equation of the parabola in the form $y = a(x - b)^2 + c$.

Check your answer first by entering your equation into Y2 and graphing both functions using TRACE. Are the two functions the same? (The arrow keys will help here. What does the up/down-arrow key do?)

Second, expand out the brackets in your function here and confirm you obtain the original function. (The coefficients you get doing this may only be approximately correct, because we truncate the numbers we take from either the table or the graph.)

Solutions

The equation of the vertical line is $x \approx 51$. On the left-hand side of the line the container is going up, on the right-hand side going down.

The axis of symmetry of the parabola is $x = b \approx 51$, so that $y \approx a(x-51)^2 + c$.

When $x = 51$, $y = c \approx 12855$, so that $c \approx 12855$. Therefore, $y \approx a(x-51)^2 + 12855$.

The coefficient of x^2 here is a, whereas in the original equation it was -4.9. Therefore, $a = -4.9$ and the equation of the parabola is $y \approx -4.9(x-51)^2 + 12855$. You could also find a by using the fact that $y(0) = 100$.

The two graphs should be the same. Check by toggling between the two curves using the up- or down-arrow key. Look at the Y values at the bottom of the screen. Try this at several points along the curves (use the left- or right-arrow key to do this).

Expanding out the brackets,

$$\begin{aligned} y &= -4.9(x-51)^2 + 12855 \\ &= -4.9(x^2 - 102x + 2601) + 12855 \\ &= -4.9x^2 + 4.9 \times 102x - 4.9 \times 2601 + 12855 \\ &= -4.9x^2 + 499.8x + 110.1 \\ &\approx -4.9x^2 + 500x + 100 \quad \text{the original equation.} \end{aligned}$$

If we used a better approximation for x, say $x \approx 51.02$ rather than 51, we obtain

$$y \approx -4.9x^2 + 499.996x + 100.102,$$

very close to the original equation.

If you go back to just before Question 11 now, turn off Y2 in [Y=] by moving the cursor over its = sign and pressing [ENTER]. Press [TRACE] to regraph Y1.