Graphics Calculator Resources for Years 9 and 10

Activity	*Compound Interest*
Year Group	9
Level	1 and 2
Strand	Number
Sub-Strand	Consumer Arithmetic
Author	Stephen Arnold, Module FM1 in *Integrating Technology in General Mathematics*, T^3 Publication, 2003. Modified by Peter McIntyre (p.mcintyre@adfa.edu.au).
Calculators	Texas Instruments TI-83 family
Description	An introduction to compound-interest calculations on a graphics calculator using formulas, tables and graphs.

Integrating Technology in General Mathematics is available at *homepages.ihug.com.au/~arnolds/t3pd.html*.

Compound Interest — Worksheet

Question 1

(a) If you invest \$5000 at an annual rate of 6% compounded annually, how much money will you have after 5 years? after 10 years?

(b) What calculation does the calculator perform each time you press ENTER (except for the first time)?

(c) Write out the calculation steps as the calculator does them to find the amount of money after 5 years. Turn this into a formula involving a number raised to power 5 and hence do the calculation on the calculator the normal way to check your answer.

Question 2

How long does it take to double your money?

(a) Make up a table of the values of N you tried and the amount of money you found with each N. Identify which N answers the question and show that it does.

(b) From the calculator table, by the end of which year does your money double?

Question 3

If you invest $5000 at 6% annual interest, compounded monthly, how long does it take to double your money?

(a) Explain how you modify the formula to allow for monthly compounding periods.

(b) In which year does the amount double now?

(c) Compare Y_1 and Y_2. What does each column represent? Which compounding method is better?

Question 4

If the annual interest rate is 8% compounded monthly, in which year does the amount double?

Question 5

By the beginning of which month of the ninth year does the amount double if the annual interest rate is 8% compounded monthly?

(a) Do we have to change the formula to answer this question?

(b) At the beginning of which month does 8.33 correspond to?

(c) Find the answer to the question from the table.

(d) In setting the WINDOW, what does X represent? Why choose 0 for Xmin? What is the smallest number we could choose for Xmax? What does Y represent? What is the smallest number we could choose for Ymax?

(e) When you use TRACE, unless you are lucky you won't find a point at which Y is exactly 10,000. This is because the cursor jumps from pixel to pixel on the screen, rather than moving smootly through all numbers. However, you can find points at which your money has at least doubled. Using the cursor, find the smallest value of X for which this is true. This is an approximation to the exact answer.

(f) If you move the cursor one pixel to the left (press the left-arrow key once) of the X value you found in (e), you can get some idea of the accuracy of your answer to the question. What are the X and Y values one pixel to the left of the X value you found in (e)? Between what times (in decimal years will do) does the exact answer then lie? You might like to think in terms like 'at this X, the Y value is just too large; at this X, the Y value is just too small'.

(g) The *intersect* operation just gives us a better approximation to the exact answer. From *intersect*, what is the answer to the question? Is it in dollars or years?

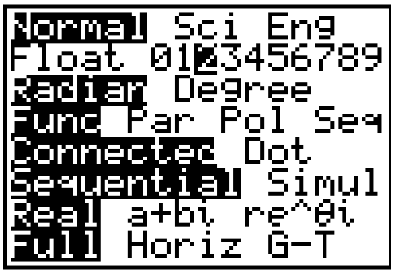
Compound Interest — Instructions

The TI-83 is a very powerful tool, more like a palm-top computer than a calculator. However, unlike Maths teachers, computers and calculators are very unforgiving. If you don't give them exactly the right information, you will probably get the wrong answer. So be careful! The process — what you do — is crucial. Compare your results with the person beside you and ask the teacher if neither of you is sure.

Question 1

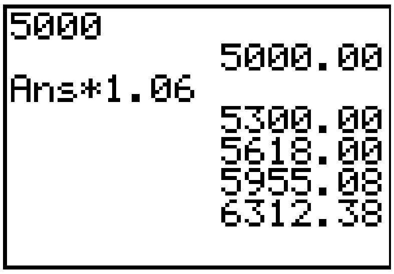
If you invest $5000 at an annual rate of 6% compounded annually, how much money will you have after 10 years?

Let's set the calculator to display numbers rounded to two decimal places since we are working in dollars and cents. Press MODE. Move the cursor to the second line, which sets the number of decimal places. Move the cursor to 2 and press ENTER. Press QUIT (2nd MODE) to return to the home screen.



Method A: Repeated Multiplication

Type	See	Result
5000 ENTER	5000	5000.00
×1.06 ENTER	Ans∗1.06	5300.00
ENTER		5618.00
ENTER		5955.08
ENTER		6312.38
⋮	⋮	⋮



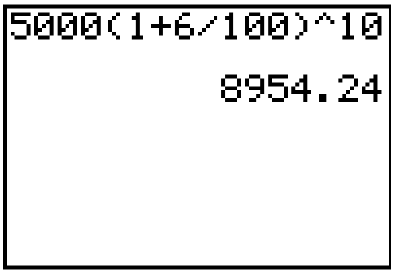
Don't forget to count the ENTER's: one ENTER = one year.

Method B: Using a Formula

The compound-interest formula is

$$A = P\left(1 + \frac{R}{100}\right)^{N},$$

where A is the amount of money after N years, P is the principal or starting amount and R is the annual interest rate expressed as a percentage. For our question, $P = 5000$, $R = 6$ and $N = 10$.



On the home screen, type the formula as

$$5000(1 + 6/100) \wedge 10$$

and press ENTER.

Question 2

How long does it take to double your money?

Clearly we could use Method A to answer this question by continuing to press ENTER until the result reaches 10,000. What about Method B? You have to find the smallest (integer) value of N that gives a value of A greater than 10,000.

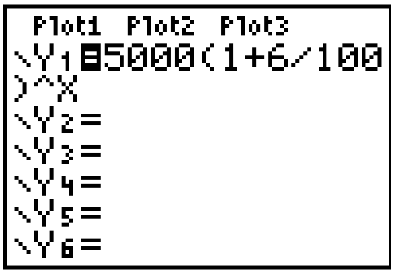
Press ENTRY (2nd ENTER). This recalls the previous command. Change the value of N to one which you think will give an answer greater than 10,000 and press ENTER to re-calculate the formula.

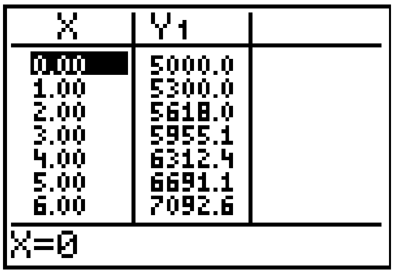
Keep guessing until you find the right value for N.

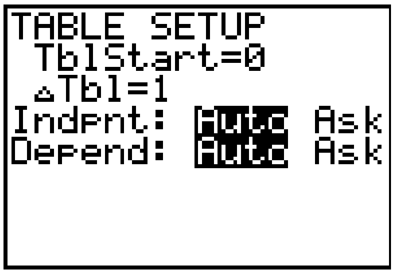
Method C: Using a Table

Press the function-definition key Y= and set Y_1 = 5000(1+6/100)∧X. Y_1 represents A, the amount of money, and X (press X,T,θ,n) represents N, the time in years.

Now press TABLE (2nd GRAPH).







If your table does not start at X = 0 and go up in steps of 1, press TBLSET (2nd WINDOW) to go to the TABLE SETUP screen or table 'WINDOW'. With the cursor and ENTER, set TblStart = 0 and ΔTbl = 1. Press 2nd GRAPH to return to the table.

Now scroll down the table until you find where Y_1 reaches 10,000. Scroll down in either column, up in the X column.

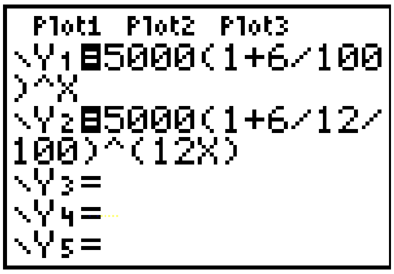
Question 3

If you invest $5000 at 6% annual interest, compounded monthly, how long does it take to double your money?

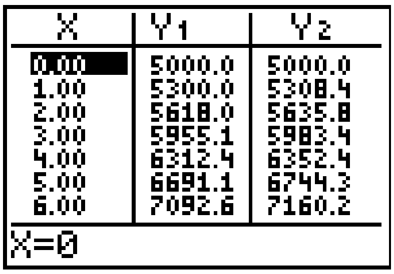
An annual interest rate of 6% compounded monthly gives a monthly interest rate R = 6/12/100, with the time now in months. The amount of money at the end of year X, month 12X, is then

$$A = 5000(1 + 6/12/100)^{12X}.$$

Press Y= and set Y_2 = 5000(1+6/12/100)∧(12X).



Now press TABLE (2nd GRAPH) and look at the values in the Y_2 column.

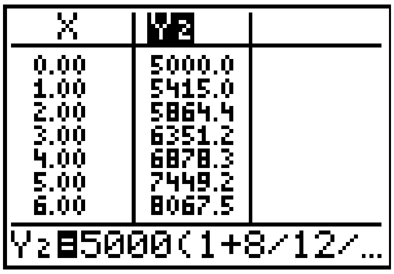


From now on, we will just use Y_2. Press [Y=], move the cursor over the = sign of Y_1 and press [ENTER] to turn it off. Press [TABLE] to return to the table.

Question 4

What if the annual interest rate is 8% compounded monthly?

Move the cursor to the heading Y_2 and press [ENTER]. Change the 6 to 8 and press [ENTER] again. The table values will reflect the new interest rate.

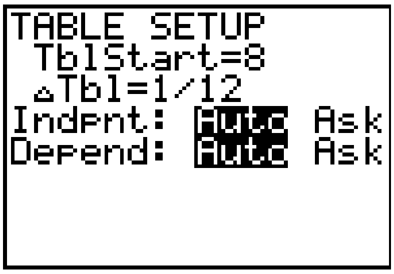


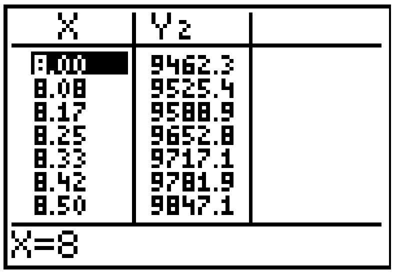
Question 5

By the beginning of which month of the ninth year does the amount double if the annual interest rate is 8% compounded monthly?

Note that X = 0.00 is the beginning of the first year, so that X = 8.00 is the beginning of the ninth year.

In TBLSET ([2nd] [WINDOW]), set TblStart = 8 and ΔTbl = 1/12 to give monthly increments. Look at the table again to answer the question.



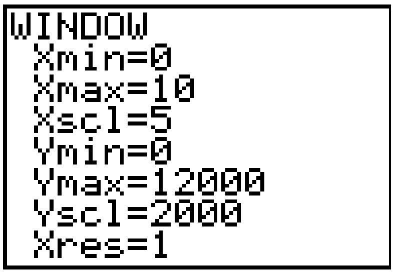


You will have to count down to find the month: 8.00 ≡ beginning of January; 8.08 ≡ beginning of February, etc.

Method D: Using a Graph

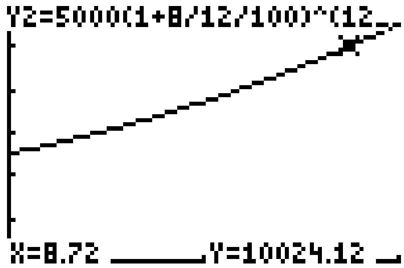
We already have the formula for the graph in Y2.

Press WINDOW. Here we have to tell the calculator how to set up the axes to view the graph. Put in the values shown on the screen to the right.

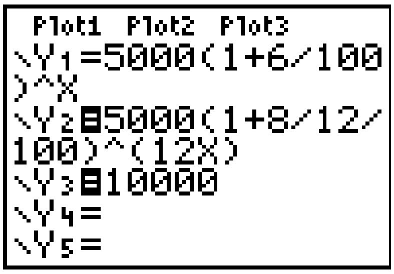


Press GRAPH to see the graph of Y2.

Press TRACE and use the left- and right-arrow keys to move along the curve.



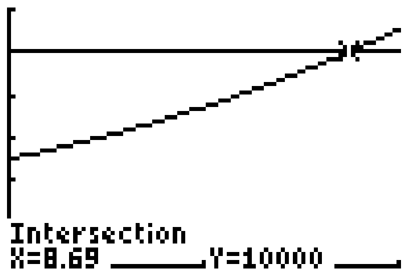
To find a more accurate answer, set Y3 = 10000, the amount of money we want to reach. Press GRAPH or TRACE to display this second curve. We will calculate the approximate intersection point of the two curves, i.e. solve the equation Y2 = Y3, to find when the original amount doubles.



Press CALC (2nd TRACE). Press 5 to select *intersect*.

The calculator now asks which curves you want to intersect (at other times, there may be more than two curves on the screen). The cursor should automatically be on Y2, the first function turned on in the function list. Press ENTER to select it. The cursor will now move to Y3. Press ENTER to select it.

The calculator now asks for a guess for the intersection point. Move the cursor somewhere near the intersection point and press ENTER.



Values are displayed to two decimal places because we set this in MODE.

Compound Interest — Teachers' Notes

Before starting the calculations here, students should have done some basic compound-interest calculations by hand. We use the calculator to be able to answer questions about compound interest that would take a long time by hand.

Some of the questions are designed to make students think about their use of the calculator as a tool. This is important, but clearly the questions can be chosen/varied to match the ability of the class.

Question 1

(a) If you invest \$5000 at an annual rate of 6% compounded annually, how much money will you have after 5 years? after 10 years?

After 5 years, you will have \$6691.13, and after 10 years, you will have \$8954.24, both rounded to the nearest cent.

(b) What calculation does the calculator perform each time you press ENTER (except for the first time)?

The calculator multiplies the previous answer/result by 1.06.

(c) Write out the calculation steps as the calculator does them to find the amount of money after 5 years. Turn this into a formula involving a number raised to power 5 and hence do the calculation on the calculator the normal way to check your answer.

The calculation is

$$5000 \times 1.06 \times 1.06 \times 1.06 \times 1.06 \times 1.06 \quad = \quad 5000 \times 1.06^5 \quad = \quad 6691.13,$$

rounded to 2 decimal places.

The calculator should not be a 'black box': students need to understand what it is actually doing.

Question 2

How long does it take to double your money?

(a) Make up a table of the values of N you tried and the amount of money you found with each N. Identify which N answers the question and explain why it does.

Clearly values for N in a table will vary, but eventually they should find that $N = 12$ does the trick. $N = 12$ is the smallest (integer) value of N for which the amount of money is greater than or equal to 10,000.

(b) From the calculator table, by the end of which year does your money double?

The amount doubles by the end of the twelfth year.

Question 3

(a) Explain how you modify the formula to allow for monthly compounding periods.

The interest rate of 6/100 per year becomes 6/12/100 per month. The time period in the exponent must now be months, giving 12X, where X is the number of years.

(b) If you invest $5000 at 6% annual interest, compounded monthly, how long does it take to double your money?

The amount still only doubles by the end of the twelfth year.

(c) Compare Y1 and Y2. What does each column represent? Which compounding option is better?

Y1 is the amount of money when the interest is compounded annually, Y2 the amount of money when the interest is compounded monthly. Compounding monthly gives a greater amount than compounding yearly at any given time, and so is the better option.

Question 4

If the annual interest rate is 8% compounded monthly, in which year does the amount double?

The amount now doubles by the end of the ninth year.

Question 5

By the beginning of which month of the ninth year does the amount double if the annual interest rate is 8% compounded monthly?

(a) Do we have to change the formula to answer this question?

No. The only thing we need to do is have the calculator table display the amounts every month rather than every year.

(b) At the beginning of which month does 8.33 correspond to?

The beginning of May.

(c) Find the answer to the question from the table.

The amount has doubled when $X = 8.75$, corresponding to the beginning of October of the ninth year. The fact that the ninth year starts when $X = 8.00$ may cause some confusion here.

(d) In setting the WINDOW, what does X represent? Why choose 0 for Xmin? What is the smallest number we could choose for Xmax? What does Y represent? What is the smallest number we could choose for Ymax?

X corresponds to time in years. Xmin is the starting time value, hence 0. Xmax must be some number larger than 9, because we know from previous work, doubling occurs in the ninth year.

Y corresponds to the amount of money in dollars. Ymax has to be some number greater than 10,000, because this is the amount we are aiming at. With some experimentation, we find that 12,000 leaves room at the top for the function formula.

(e) When you use TRACE, unless you are lucky you won't find a point at which Y is exactly 10,000. This is because the cursor jumps from pixel to pixel on the screen, rather than moving smootly through all numbers. However, you can find points at which your money has at least doubled. Using the cursor, find the smallest value of X for which this is true. This is an approximation to the exact answer.

Using TRACE, the smallest value of X for which $Y \geq 10{,}000$ *is* $X = 8.72$.

(f) If you move the cursor one pixel to the left (press the left-arrow key once) of the X value you found in (e), you can get some idea of the accuracy of your answer to the question. What are the X and Y values one pixel to the left of the X value you found in (e)? Between what times (in decimal years will do) does the exact answer then lie? You might like to think in terms like 'at this X, the Y value is just too large; at this X, the Y value is just too small'.

For the pixel one to the left of the X value in (e), we have $X = 8.62$ *and* $Y = 9939.45$. *The exact X value (time in years) therefore must lie between 8.62* ($Y < 10{,}000$) *and 8.72* ($Y > 10{,}000$).

(g) The *intersect* operation just gives us a better approximation to the exact answer. From *intersect*, what is the answer to the question? Is it in dollars or years?

According to intersect, $Y = 10{,}000$ *when* $X = 8.69$ *(rounded to 2 decimal places). This is in September, so the answer still remains 'by the beginning of October'.*

There are at least two more methods we could use on the TI-83 to solve the above problems: using the Solver (MATH 0), which finds zeros of any function; and using the TVM Solver, a special finance version of the Solver found on the x^{-1} key (TI-83) or in the Finance App (TI-83+).

If you want to do calculations with regular repayments, the TVM Solver is the method to use, because the formulas become more complicated. Make sure you read the instructions carefully, especially those on what sign to use for the various amounts.