Graphics Calculator Resources for Years 9 and 10

Activity	*A Classic Problem — The Hare and Tortoise*
Year Group	10
Level	1
Strand	Algebra
Sub-Strand	Sketching Other Graphs, Simultaneous Equations
Author	This version comes from Ruth Hubbard of QUT. Modified by Peter McIntyre (p.mcintyre@adfa.edu.au).
Calculators	TI-83 family
Description	The graphs of the distances covered versus time in this classic race are used to answer various questions about the race, such as who won and by how much. A fun exercise in putting questions into maths and solving equations graphically.

A Classic Problem

A hare and tortoise compete in a one-kilometre race. The distance each competitor has travelled from the starting point is given by a formula. In time t **minutes**, the distance in **metres** travelled by the hare is given by $H(t) = \frac{500}{3}(2\sqrt{t} + \sqrt[3]{t})$, while the distance in **metres** travelled by the tortoise is given by $T(t) = 100t + 250\sqrt{t}$.

Press [Y=] and enter the formulas for H and T. You have to use X ([X,T,θ,n]) as the independent variable. The cube root is [MATH] [4].

Set your [WINDOW] so that the two graphs go from the bottom left to the top right of the screen. *Hints*: The race takes about 5 minutes. How far is the race?

If you select *Simul* in the [MODE] menu of your TI-83 before graphing, you will get a real-time view of the race. Choosing the graph style of a circle with a tail[1] for each competitor makes it even better.

Answer the following questions, writing down the steps you took. Plotting the lines Y3 = 500 and Y4 = 1000 and using *intersect* in the [CALC] menu will be helpful. You may need to increase *Ymax* when using *intersect* so that the function formulas do not obscure the point you are interested in.

1. Who gets to the halfway point first? How long does it takes them? Verify your answer algebraically.

2. What is the time and distance at which the two runners are neck and neck?

3. Who wins the race, by what time margin and by what distance margin?

The printed version of this activity contains a picture here. Unfortunately we cannot include it in the web version for copyright reasons. Unfortunately also, we can no longer find it on the web.

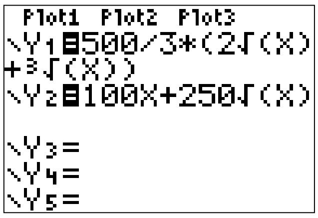
However, there is a suitable picture at *http://www.jimnuttle.com/illo1/torhar.gif*, which you might like to include if you download this activity from the web.

[1] Press [Y=], move the cursor to the left of Y1 and press [ENTER] a couple of times. Repeat for Y2.

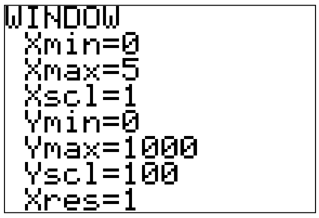
Teacher's Notes

The questions in this version have been written in general terms deliberately for a good class. For a less-advanced class, students may need to be led a little through each question. For example: *What equation do we need to solve to answer this question? What does this mean about the graphs of each side of the equation? How do we solve this equation on the calculator?* and so on.

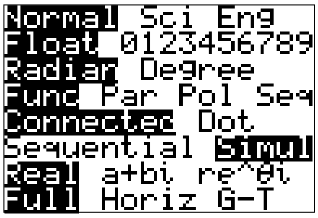
Press Y= and put the equation for the hare in Y_1 and that for the tortoise in Y_2. Watch brackets here. You might like to discuss with the class how to write the formulas in a suitable form for the calculator. Time t becomes X on the calculator.



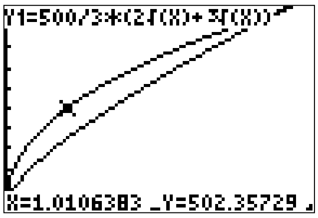
Then set the WINDOW. Discuss first with the class what each axis represents and suitable scales. The Y axis is distance in metres, so $0 < \mathrm{Y} < 1000$. The winner is then the competitor whose graph first reaches the top of the screen (providing *Simul* is set in MODE).



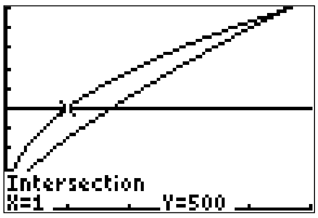
The time (X) scale has to be guessed. The race takes a little less than 5 minutes, so $0 < \mathrm{X} < 5$ gives a good view. Set *Xscl*, the distance between the tick marks on the X axis to 1 and *Yscl* to 100. If either of these is too small, you will get a double line for the axis.



1. From the graph (use TRACE and the up/down arrows to see which graph is which), the hare clearly reaches the halfway point (500 m) first.



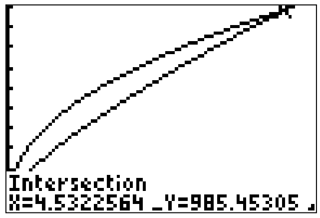
To find how long the hare took, solve $H(t) = 500$ for t: set $Y_3 = 500$ and find the intersection of Y_1 and Y_3 using *intersect* in the CALC menu.
After you select *intersect*, the cursor will be on Y_1: press ENTER to select it. The cursor will move to Y_2: press the down-arrow key to move it to Y_3 and press ENTER to select it. Move the cursor to somewhere near the intersection and press ENTER to provide a guess.



The value for t is 1 minute, a value we can confirm algebraically to be exact by substituting $t = 1$ into the equation for the hare. Note that it is easy to **verify** that $t = 1$ is a solution, but tricky to **solve** $H(t) = 500$ algebraically (it turns into a cubic equation).

The hare reaches the halfway point first in a time of 1 minute.

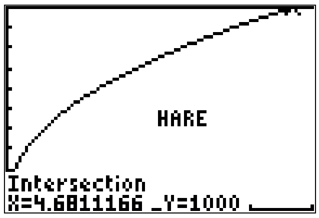
2. To find when they are neck and neck, we have to solve $H(t) = T(t)$, that is find the intersection of Y_1 and Y_2 (algebraically, this turns into a quartic equation). We obtain, using *intersect*, $t = 4.53$ minutes and distance equal to 985 m, both accurate to 3 significant digits. It might be useful to *Zoom In* ([ZOOM] [2]) on this part of the graph to see the two curves more clearly.

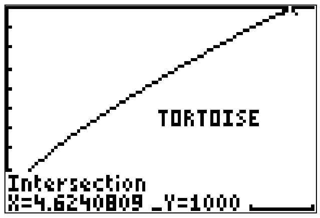


The hare and tortoise are neck and neck after about 4.53 minutes or about 4 minutes 32 seconds, at a distance of about 985 metres from the start.

3. To find the winner, we have to determine the time at which each competitor reaches the finish (1000 m).

Setting $Y_4 = 1000$, we find the hare finishes at $t =$ 4.681 minutes (intersection of Y_1 and Y_4) and the tortoise finishes at $t = 4.624$ minutes (intersection of Y_2 and Y_4).





To find the distance margin, calculate $H(4.624)$, the position of the hare when the tortoise finishes: $H(4.624) \equiv Y_1(4.624) = 994.45$ m, rounded to 5 significant digits.

The tortoise wins the race by a margin of 0.057 minutes or 3.42 seconds. The distance margin is 5.55 m.

Using the Newton-Raphson Method

The problems here are also a good application of the Newton-Raphson method for finding the zero of a function. For this, it is useful to have a program.

The program NEWTON for the TI-83/83+ can be downloaded at *www.ma.adfa.edu.au* under *High School and College Activities*. The program finds the zero of a graphed function, given an initial guess provided by the position of the cursor on the graph.

The program finds zeroes of the function in Y_1, so it is useful to put the hare and tortoise equations in Y_2 and Y_3. Y_1 can then be defined as $Y_1 = Y_2 - 500$ to find when the hare reaches halfway; $Y_1 = Y_2 - Y_3$ to find when they are neck and neck; $Y_1 = Y_2 - 1000$ to find when the hare finishes and $Y_1 = Y_3 - 1000$ to find when the tortoise finishes.