Graphics Calculator Resources for Years 9 and 10

Activity	*A Classic Problem — The Hare & Tortoise*
Year Group	10
Level	1
Strand	Algebra
Sub-Strand	Sketching Other Graphs, Simultaneous Equations
Author	This version comes from Ruth Hubbard of QUT. Modified by Peter McIntyre (p.mcintyre@adfa.edu.au).
Calculators	CFX-9850 family
Description	The graphs of the distances covered versus time in this classic race are used to answer various questions about the race, such as who won and by how much. A fun exercise in putting questions into maths and solving equations graphically.

A Classic Problem

A hare and tortoise compete in a one-kilometre race. The distance each competitor has travelled from the starting point is given by a formula. In time t **minutes**, the distance in **metres** travelled by the hare is given by $H(t) = \frac{500}{3}(2\sqrt{t} + \sqrt[3]{t})$, while the distance in **metres** travelled by the tortoise is given by $T(t) = 100t + 250\sqrt{t}$.

Press [MENU] [5] (GRAPH) and enter the formulas for H and T in Y1 and Y2 respectively. You have to use X ([X,θ,T]) as the independent variable. The cube root is [SHIFT] [(].

Set your *View Window* ([SHIFT] [F3]) so that the two graphs will go from the bottom left to the top right of the screen. *Hints*: The race takes about 5 minutes. How far is the race?

If you select *Simul Graph On* in the [SET UP] menu of your calculator before graphing, you will get a real-time view of the race.

Answer the following questions, writing down the steps you took. [Trace], *ISCT* (intersection), *Y·CAL* (find y given x) and *X·CAL* (find x given y) in the [G-Solv] menu will be helpful. Press the right arrow after any of these operations to find further values.

1. Who gets to the halfway point first? How long does it takes them? Verify your answer algebraically.

2. What is the time and distance at which the two runners are neck and neck?

3. Who wins the race, by what time margin and by what distance margin?

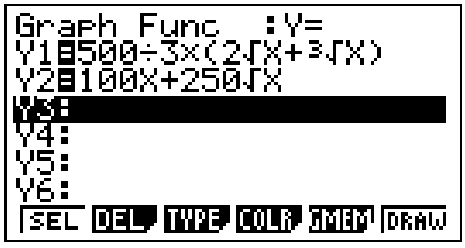
The printed version of this activity contains a picture here. Unfortunately we cannot include it in the web version for copyright reasons. Unfortunately also, we can no longer find it on the web.

However, there is a suitable picture at *http://www.jimnuttle.com/illo1/torhar.gif*, which you might like to include if you download this activity from the web.

Teacher's Notes

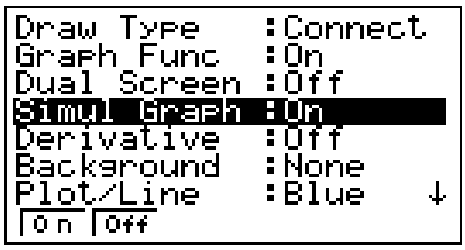
The questions in this version have been written in general terms deliberately for a good class. For a less-advanced class, students may need to be led a little through each question. For example: *What equation do we need to solve to answer this question? What does this mean about the graphs of each side of the equation? How do we solve this equation on the calculator?* and so on.

Press [MENU] [5] (GRAPH) and put the equation for the hare in Y1 and that for the tortoise in Y2. You might like to discuss with the class how to write the formulas in a suitable form for the calculator. Time t becomes X on the calculator.



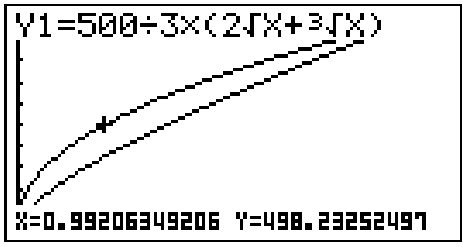
Then set the *View Window*: discuss first with the class what each axis represents and suitable scales. The Y axis is distance in metres, so $0 <$ Y < 1000. The winner is then the competitor whose graph first reaches the top of the screen (assuming *Simul Graph On*).

The time (X) scale has to be guessed. The race takes a little under 5 minutes, so $0 <$ X < 5 gives a good view.
Press [EXIT] to return to the *Graph Func* screen.

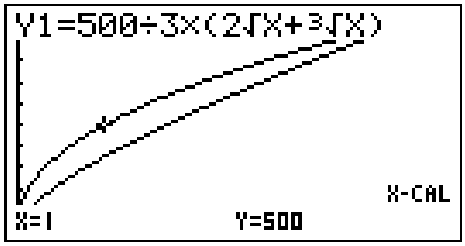


1. Press [F6] (DRAW), then [F1] (Trace): use the up/down arrows to see which graph is which.

The hare clearly reaches the halfway point (500 m) first.



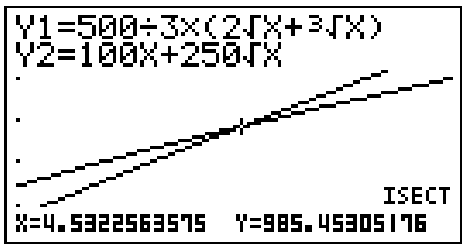
To find how long the hare took, solve $H(t) = 500$ for t using *X·CAL* ([G-Solv] [F6] [F2]). Select the appropriate curve (Y1) using the up/down arrow keys and press [EXE].
Type in the Y value (500) and press [EXE] again.



The value for t is 1 minute, a value we can confirm algebraically to be exact by substituting $t = 1$ into the equation for the hare. Note that it is easy to *verify* that $t = 1$ is a solution, but tricky to *solve* $H(t) = 500$ algebraically (it turns into a cubic equation).

The hare reaches the halfway point first in a time of 1 minute.

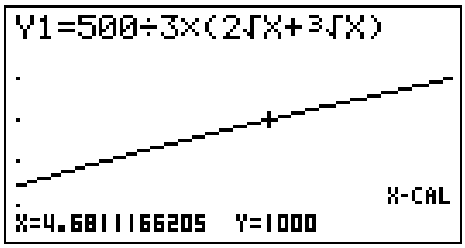
2. To find when they are neck and neck, we have to solve $H(t) = T(t)$, that is find the intersection (*ISCT*) of Y1 and Y2 (algebraically, this turns into a quartic equation). We obtain $t = 4.53$ minutes and a distance of 985 m, both accurate to 3 significant digits. It might be useful to *Zoom IN* (F2 F3) on this part of the graph (as we did for the figure) to see the two curves more clearly.

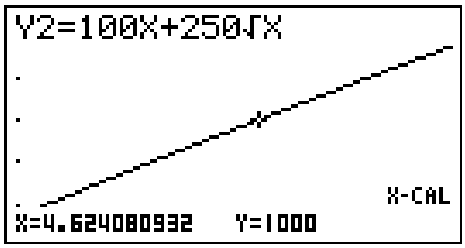


The hare and tortoise are neck and neck after about 4.53 minutes or about 4 minutes 32 seconds, 985 metres from the start.

3. To find the winner, we have to determine the time at which each competitor reaches the finish (1000 m).

Choosing the appropriate curve with the arrow keys (or graphing one curve at a time) and using *X·CAL*, we find the hare finishes at $t = 4.681$ minutes and the tortoise finishes at $t = 4.624$ minutes.





To find the distance margin, calculate $H(4.624)$, the position of the hare when the tortoise finishes, using *Y·CAL*: $H(4.624) = 994.45$ m, rounded to 5 significant digits.

The tortoise wins the race by a margin of 0.057 minutes or 3.42 seconds. The distance margin is 5.55 m.