Graphics Calculator Resources for Years 9 and 10

Activity	*A Classic Problem — The Hare and Tortoise*
Year Group	10
Level	1
Strand	Algebra
Sub-Strand	Sketching Other Graphs, Simultaneous Equations
Author	This version comes from Ruth Hubbard of QUT. Modified by Peter McIntyre (p.mcintyre@adfa.edu.au).
Calculators	Sharp EL-9650/9900
Description	The graphs of the distances covered versus time in this classic race are used to answer various questions about the race, such as who won and by how much. A fun exercise in putting questions into maths and solving equations graphically.

A Classic Problem

A hare and tortoise compete in a one-kilometre race. The distance each competitor has travelled from the starting point is given by a formula. In time t **minutes**, the distance in **metres** travelled by the hare is given by $H(t) = \frac{500}{3}(2\sqrt{t} + \sqrt[3]{t})$, while the distance in **metres** travelled by the tortoise is given by $T(t) = 100t + 250\sqrt{t}$.

Press [Y=] and enter the formulas for H and T in Y1 and Y2 respectively. You have to use X ([X/θ/T/n]) as the independent variable. The cube root is [3] [2ndF] [a^b].

Set your [WINDOW] so that the two graphs go from the bottom left to the top right of the screen. *Hints*: The race takes about 5 minutes. How far is the race?

The race looks better if the two graphs are plotted simultaneously. Ask your teacher how to do this.

Answer the following questions, writing down the steps you took. You will need to plot the lines Y3 = 500 and Y4 = 1000 and use *Intsct* in the CALC menu. You may need to increase *Ymax* when using *Intsct* so that you can see better the point you are interested in.

1. Who gets to the halfway point first? How long does it takes them? Verify your answer algebraically.
2. What is the time and distance at which the two runners are neck and neck?
3. Who wins the race, by what time margin and by what distance margin?

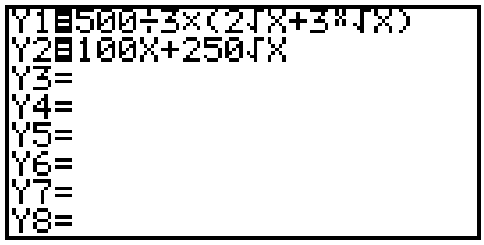
The printed version of this activity contains a picture here. Unfortunately we cannot include it in the web version for copyright reasons. Unfortunately also, we can no longer find it on the web.

However, there is a suitable picture at *http://www.jimnuttle.com/illo1/torhar.gif*, which you might like to include if you download this activity from the web.

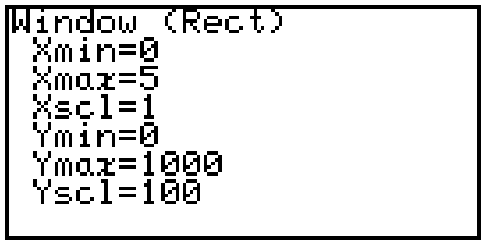
Teacher's Notes

The questions in this version have been written deliberately in general terms for a good class. For a less-advanced class, students may need to be led a little through each question. For example: *What equation do we need to solve to answer this question? What does this mean about the graphs of each side of the equation? How do we solve this equation on the calculator?* and so on.

Press [Y=] and put the equation for the hare in Y1 and that for the tortoise in Y2. Be careful to put in the × before the bracket in Y1. You might like to discuss with the class how to write the formulas in a suitable form for the calculator. Time t becomes X on the calculator.

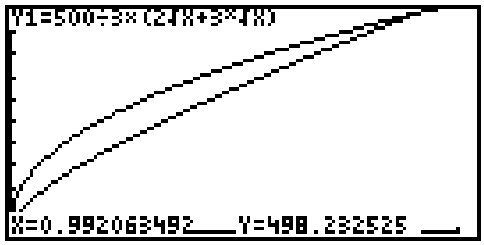


Then set the [WINDOW]. Discuss first what each axis represents and suitable scales. The Y axis represents distance in metres, so $0 < Y < 1000$. The winner is then the competitor whose graph first reaches the top of the screen (providing the graphs are plotted simultaneously — see below).

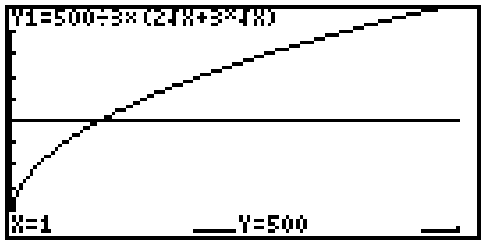


The time (X) scale has to be guessed. The race takes a little less than 5 minutes, so (after experimenting) $0 < X < 5$ gives a good view. Set *Xscl*, the distance between the tick marks on the X axis to 1 and *Yscl* to 100. If either of these is too small, you will get a double line for the axis.

1. Press [TRACE]; use the up/down arrows to see which graph is which — you need EXPRES ON in [FORMAT] ([2ndF] [ZOOM]) for this.
   The hare clearly reaches the halfway point (500 m) first.

   To find how long the hare took, solve $H(t) = 500$ for t: set Y3 = 500 and find the intersection of Y1 and Y3 using *Intsct* in the [CALC] menu.

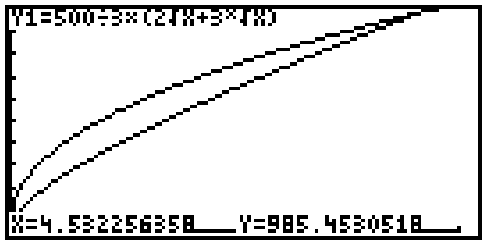
   

   When you select *Intsct*, the calculator will find one of the intersections on screen. Keep selecting *Intsct* until you find the intersection you want. For the figure, we have turned off Y2.

The value for t is 1 minute, a value we can confirm algebraically to be exact by substituting $t = 1$ into the equation for the hare. Note that it is easy to **verify** that $t = 1$ is a solution, but tricky to **solve** $H(t) = 500$ algebraically (it turns into a cubic equation).

The hare reaches the halfway point first in a time of 1 minute.

2. To find when they are neck and neck, we have to solve $H(t) = T(t)$, that is find the intersection of Y1 and Y2 (algebraically, this turns into a quartic equation).

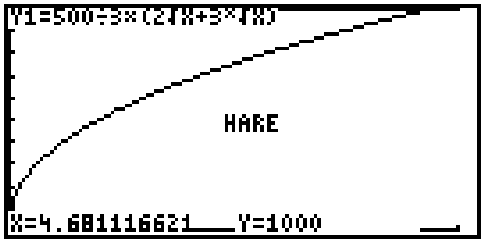


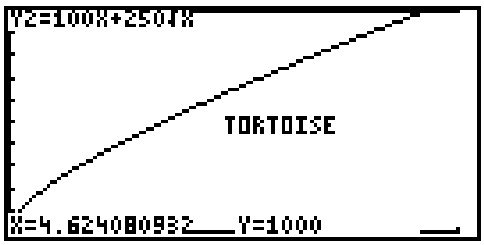
Because of the way *Intsct* works, it is probably a good idea to turn off any functions not involved (cursor on the = sign in Y= and press ENTER).

We obtain, using *Intsct* (twice), $t = 4.53$ minutes and distance equal to 985 m, both accurate to 3 significant digits. It might be useful to *Zoom In* on this part of the graph to see the two curves more clearly.

The hare and tortoise are neck and neck after about 4.53 minutes or about 4 minutes 32 seconds, at a distance of about 985 metres from the start.

3. To find the winner, we have to determine the time at which each competitor reaches the finish (1000 m).





Setting Y4 = 1000, we find the hare finishes at $t = 4.681$ minutes (intersection of Y1 and Y4) and the tortoise finishes at $t = 4.624$ minutes (intersection of Y2 and Y4).

To find the distance margin, calculate $H(4.624)$, the position of the hare when the tortoise finishes: $H(4.624) \equiv$ Y1 $(4.624) = 994.45$ m, to 5 significant digits.
Y1 is in the VARS (2ndF X/θ/T/n) EQVARS XY menu.

The tortoise wins the race by a margin of 0.057 minutes or 3.42 seconds. The distance margin is 5.55 m.

Setting simultaneous graph plotting

The only way to set simultaneous graph plotting seems to be via a program. Press PRGM, select NEW and call the program SIMUL. Press PRGM again, then F and scroll down to find *Simul*. Press ENTER to select it. This is your program.

Press the Home key (the one with +, −, × and ÷ on it), press PRGM, select EXEC, scroll down to your program and press ENTER to run it. Graphs will now plot simultaneously.

To go back to sequential graphing, use the corresponding program SEQUEN, containing the single command *Sequen*, or reset defaults in OPTION C.