Graphics Calculator Resources for Years 9 and 10

Activity	*Probably Finding* π
Year Group	10
Level	1 and 2
Strand	Chance and Data
Sub-Strand	Probability
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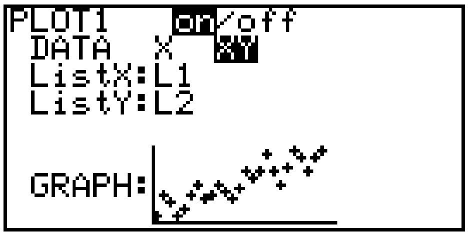
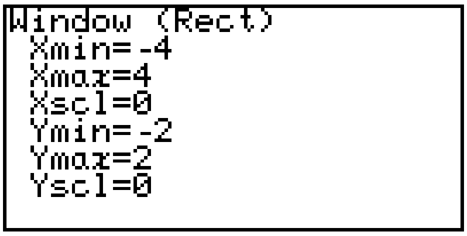
Calculators	Sharp EL-9650/9900
Description	An experimental-probability method for finding π.

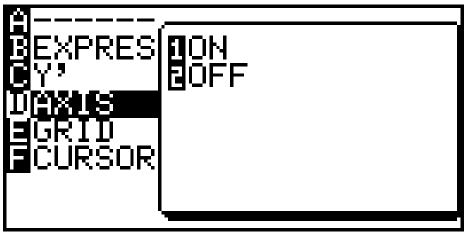
Probably Finding π

1. **Set up the plot screen.** Press STATPLOT and select PLOT1. Set up PLOT1 as shown using the cursor and ENTER.
   L1 and L2 are 2ndF 1 and 2ndF 2 respectively.
   Press STATPLOT again to access the GRAPH options: here we want G (S.D.) 2.





2. **Set a WINDOW.** Press WINDOW and set up as shown so that $-4 < X < 4$ and $-2 < Y < 2$.
   Make sure all the functions in Y= are turned off.



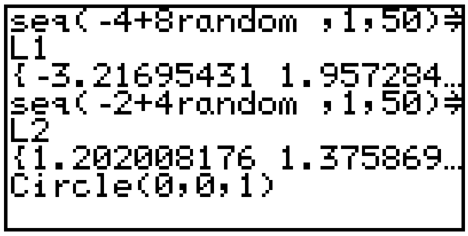
3. **Turn off the axes.** Press FORMAT (2ndF ZOOM), then D 2 to select AXIS OFF. Press the Home key (the one with +, −, ×, ÷ on it) to return to the home screen.

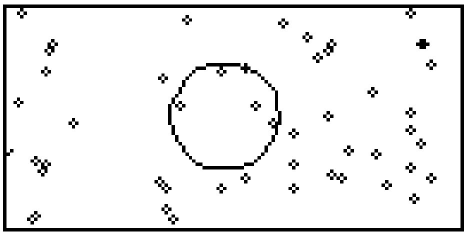
4. **Store the coordinates of 50 random points and draw a unit circle.**

   Type in and execute the following commands.

   seq $(-4 + 8\text{random}, 1, 50) \Rightarrow$ L1
   seq $(-2 + 4\text{random}, 1, 50) \Rightarrow$ L2
   Circle $(0, 0, 1)$

   Seq: in LIST (2ndF STATPLOT) OPE.
   random: in MATH PROB.
   Circle: in DRAW (2ndF WINDOW) DRAW.
   The arrow represents the STO key.





   How many points landed inside the circle? If a point lands on the circle, toss a coin to decide if it is in or out.

5. Answer the following questions to find an estimate for π.

(a) What is the area of the WINDOW?

(b) What is the area of the circle?

(c) What is the theoretical probability of a random point in the WINDOW landing in the circle? *Hint:* Think areas.

(d) What is the experimental probability? *Hint:* How many points are there in the WINDOW? How many of these lie in the circle?

(e) Assuming the experimental probability and the theoretical probability are approximately equal, solve for an estimate of π.

Pool a series of results to obtain a better estimate for π.

Hint: To do further runs, repeatedly press [ENTRY] ([2nd] [Exp]) to recall the commands in the appropriate order and press [ENTER] to re-execute them. Each time you do this, you will have a different set of data.

An even better way is to type the two *Seq* commands and the *Circle* command all on one line, separated by semi-colons ([ALPHA] [Exp]). After a plot, press the Home key and then [ENTRY] and [ENTER] to re-execute all three commands in one go to obtain a new plot. Doing this repeatedly produces lots of data rapidly.

When you have finished, turn your axes on again in [FORMAT] and clear lists L1 and L2 as follows: press [STAT] EDIT [ENTER]; move the cursor to the list heading and press [DEL] [ENTER].

Note for Teachers

Calculator operations

The x coordinates of each point generated are stored in list L1 by the first *Seq* command. The *random* command generates a random number[1] between (but not equal to) 0 and 1. Therefore, *8random* generates a random number between 0 and 8, and $-4+8random$ a random number between -4 and 4, the range of x values in WINDOW.

Similarly, the second *Seq* command, containing $-2+4random$, generates random numbers between -2 and 2 for the y coordinate and stores them in list L2.

The command Circle(x, y, r) draws a circle, centre (x, y) and radius r, on the screen.

Question 5

(a) The area of the WINDOW is $8 \times 4 = 32$.

(b) The area of the circle is $\pi \times 1^2 = \pi$.

(c) The theoretical probability of a random point in the WINDOW landing in the circle is the ratio of the area of the circle to the area of the WINDOW, i.e. $\pi/32$.

(d) The experimental probability is the ratio of the number of points (N say) in the circle to the total number of points in the WINDOW, i.e. $N/50$.

(e) If we assume the experimental probability and the theoretical probability are approximately equal, we have

$$\frac{\pi}{32} \approx \frac{N}{50},$$

so that

$$\pi \approx \frac{32N}{50} = 0.64N.$$

Counting N then gives us an estimate for π.

It's a good idea to pool the data from all the students to obtain (hopefully) a better estimate for π than individual students will obtain. You could discuss why more data should give a better estimate (experimental probability $\rightarrow$ theoretical probability as the number of data points $\rightarrow \infty$).

If you pool the data, find the mean number of points that land in the circle and multiply by 0.64 to find the mean estimate for π. It is easier to average integers than to average all the individual estimates for π.

[1] If the calculators have all been reset before this activity, successive *rand* commands will generate the same sequence of random numbers on each calculator, so that students will have identical results. If this is the case, ask each student to store his/her favourite number between 1 and 100 in *random* (key strokes: *number* [STO] [MATH] PROB [1] [ENTER]), so that they each start with different random-number seeds and therefore obtain different sequences of random numbers.

You can gain some idea of the expected accuracy in your estimate for π by using the mean $N \pm$ one standard deviation as values for N in the formula. Does the actual value of π lie in this range?

The hint on repeated plots allows individual students to generate lots of data of their own if they wish.

Extension

How would you get the calculator to decide whether a point (x, y) lies inside the circle?

A point (x, y) *lies inside the circle if* $\sqrt{x^2 + y^2} < 1$ *or, equivalently, if* $x^2 + y^2 < 1$.

Write a calculator or computer program to generate the random points, calculate the number lying inside the circle and hence estimate π automatically.

Further Research

Find out about Buffon's needle problem, a 'hands-on' forerunner of this problem.